

ALGEBRA

Q) If $(x - \frac{1}{x}) = \sqrt{6}$ and $x > 1$, what is the value of $(x^8 - \frac{1}{x^8})$?

यदि $(x - \frac{1}{x}) = \sqrt{6}$ और $x > 1$ है, तो $(x^8 - \frac{1}{x^8})$ का मान कितना है?

$$\begin{aligned} \downarrow \\ x + \frac{1}{x} &= \sqrt{6+4} \\ &= \sqrt{10} \end{aligned}$$

$$x^2 + \frac{1}{x^2} = 10 - 2 = 8$$

$$x^4 + \frac{1}{x^4} = 8^2 - 2 = 62$$

$$\downarrow \\ (x - \frac{1}{x})(x + \frac{1}{x})(x^2 + \frac{1}{x^2})(x^4 + \frac{1}{x^4})$$

$$\sqrt{6} \times \sqrt{10} \times 8 \times 62$$

$$496 \sqrt{60}$$

$$4 \times 15$$

$$496 \times 2 \sqrt{15}$$

$$992 \sqrt{15}$$

Q) If $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$, then $(x^2 + \frac{1}{x^2})$ is equal to:

यदि $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$ है, तब $(x^2 + \frac{1}{x^2})$ बराबर है:

$$\frac{(\sqrt{x})^2 + \frac{1}{(\sqrt{x})^2}}{(\sqrt{x})^2} = (\sqrt{6})^2 - 2$$

$$x + \frac{1}{x} = 4$$

$$x^2 + \frac{1}{x^2} = 4^2 - 2 = 14$$

Q) If $\sqrt{x} - \frac{1}{\sqrt{x}} = 4$, then $x^2 + \frac{1}{x^2}$ is equal to.

यदि $\sqrt{x} - \frac{1}{\sqrt{x}} = 4$ है, तब $x^2 + \frac{1}{x^2}$ बराबर है।

$$\frac{(\sqrt{x})^2 + \frac{1}{(\sqrt{x})^2}}{(\sqrt{x})^2} = 4^2 + 2$$

$$x + \frac{1}{x} = 18$$

$$x^2 + \frac{1}{x^2} = 18^2 - 2$$

$$322$$

Q) If $\sqrt{x} + \frac{1}{\sqrt{x}} = 3$, then the value of $x^3 + \frac{1}{x^3}$.

यदि $\sqrt{x} + \frac{1}{\sqrt{x}} = 3$ है, तो $x^3 + \frac{1}{x^3}$ का मान ज्ञात कीजिए।

$$x + \frac{1}{x} = 3^2 - 2$$

$$x^3 + \frac{1}{x^3} = 7^3 - 3 \times 7$$

$$343 - 21$$

$$322$$

Q) If $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{3}$, then what is the value of $(x^4 + \frac{1}{x^4})$?

यदि $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{3}$ है, तब $(x^4 + \frac{1}{x^4})$ का मान क्या है?

$$x + \frac{1}{x} = (\sqrt{3})^2 + 2$$

$$x + \frac{1}{x} = 5$$

$$x^2 + \frac{1}{x^2} = 5^2 - 2 = 23$$

$$x^4 + \frac{1}{x^4} = 23^2 - 2 = 527$$

CONJUGATE

$$\# x = a + b$$

$$\frac{1}{x} = \frac{1}{a+b} = a-b$$

where

$$a^2 - b^2 = 1$$

&

$$a > b$$

Ex: $x = 7 + 4\sqrt{3}$

$$\frac{1}{x} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{49-48} = 7-4\sqrt{3}$$

①

$$\bullet x = 2 - \sqrt{3}$$

$$\frac{1}{x} = 2 + \sqrt{3}$$

Q) If $x = 7 - 4\sqrt{3}$, then find the value of $(x^2 + \frac{1}{x^2})$

यदि $x = 7 - 4\sqrt{3}$ है, तो $(x^2 + \frac{1}{x^2})$ का मान ज्ञात कीजिए।

$$+ \frac{1}{x} = 7 + 4\sqrt{3}$$

$$x + \frac{1}{x} = 7 + 7 = 14$$

$$x^2 + \frac{1}{x^2} = 14^2 - 2 = 194$$

Q) If $x = (\sqrt{6} - 1)^{\frac{1}{3}}$, the value of $(x - \frac{1}{x})^3 + 3(x - \frac{1}{x})$ is,

यदि $x = (\sqrt{6} - 1)^{\frac{1}{3}}$ है, $(x - \frac{1}{x})^3 + 3(x - \frac{1}{x})$ का मान है।

$$x^3 = \sqrt{6} - 1$$

$$\frac{1}{x^3} = \frac{1}{\sqrt{6} - 1} \times \frac{\sqrt{6} + 1}{\sqrt{6} + 1}$$

$$\frac{\sqrt{6} + 1}{\sqrt{6} - 1} = \frac{\sqrt{6} + 1}{5}$$

$$\sqrt{6} - 1 - \frac{\sqrt{6} + 1}{5}$$

$$5\sqrt{6} - 5 - \sqrt{6} - 1$$

$$\frac{4\sqrt{6} - 6}{5}$$

$$\begin{aligned} x^3 - \frac{1}{x^3} - 3 \times 1 \left(x - \frac{1}{x} \right) + 3 \left(x - \frac{1}{x} \right) \\ = x^3 - \frac{1}{x^3} \end{aligned}$$

Q) If $x + \frac{1}{x} = 1$, then the value of $\frac{x^2 + 7x + 1}{x^2 + 11x + 1}$ is

यदि $x + \frac{1}{x} = 1$ है, तो $\frac{x^2 + 7x + 1}{x^2 + 11x + 1}$ का मान क्या है?

Divide (N) & (D) by x

$$\frac{x + \frac{1}{x} + 7}{x + \frac{1}{x} + 11}$$

$$= \frac{1 + 7}{1 + 11} = \frac{8}{12} = \frac{2}{3}$$

Q) If $x - \frac{1}{x} = 5$, $x \neq 0$, then the value of $\frac{x^6 - 5x^3 - 1}{x^6 + 7x^3 - 1}$ is

यदि $x - \frac{1}{x} = 5$, $x \neq 0$ है, तो $\frac{x^6 - 5x^3 - 1}{x^6 + 7x^3 - 1}$ का मान ज्ञात कीजिए।

$$\begin{aligned} x^3 - \frac{1}{x^3} &= 5^3 + 3 \times 5 \\ &= 140 \end{aligned}$$

Divide (N) & (D) by x^3

$$\frac{x^3 - \frac{1}{x^3} - 5}{x^3 - \frac{1}{x^3} + 7}$$

$$\frac{140-5}{140+7} = \frac{-135}{147} = \frac{45}{49}$$

* Case 1:-

if $x + \frac{1}{x} = 2$

then $x = 1$

also $x^m + \frac{1}{x^m} = 2$

$$\begin{aligned} x^2 + 1 &= 2x \\ x^2 - 2x + 1 &= 0 \\ &\quad +1 \quad -1 \end{aligned}$$

* Case 2:-

if $x + \frac{1}{x} = -2$

$x = -1$

* Case 3:-

if $x + \frac{1}{x} = 1 \rightarrow (x+1)(x^2-x+1) = 0 \times (x+1)$

then $x^3 = -1$

$$x^3 + 1 = 0$$

$x^3 = -1$

* Case 4:-

if $x + \frac{1}{x} = -1 \Rightarrow (x-1)(x^2+x+1) = 0 \times (x-1)$

$x^3 = +1$

$$x^3 - 1 = 0$$

$x^3 = +1$

* Case 5:-

$x + \frac{1}{x} = \pm \sqrt{3}$

$x^6 = -1$

TYPE-V

Q) If $x + \frac{1}{x} = 2$, then find the value of $\left(x^4 + \frac{4}{x^4}\right)\left(x^3 - \frac{4}{x^3}\right)$

यदि $x + \frac{1}{x} = 2$ है, तो $\left(x^4 + \frac{4}{x^4}\right)\left(x^3 - \frac{4}{x^3}\right)$ का मान ज्ञात करें।

$x = 1$

$\left(1 + \frac{4}{1}\right)\left(1 - \frac{4}{1}\right)$

$5 \times (-3)$

$= -15$

Q) If $x + \frac{1}{x} = 2$, then the value of $x^{100} + \frac{1}{x^{100}}$ is.

यदि $x + \frac{1}{x} = 2$ है, तो $x^{100} + \frac{1}{x^{100}}$ का मान है।

(2)

Q) If $x + \frac{1}{x} = 2$ and x is real, then the value of $x^{17} + \frac{1}{x^{17}}$,

यदि $x + \frac{1}{x} = 2$ और x वास्तविक है, तो $x^{17} + \frac{1}{x^{17}}$ का मान है।

$x = 1$

(2)

Q) If $x + \frac{1}{x} = 2$, then find the value of $x^{1823} + \frac{1}{x^{1823}}$

यदि $x + \frac{1}{x} = 2$ है, तो $x^{1823} + \frac{1}{x^{1823}}$ का मान ज्ञात कीजिए।

$x = 1$

(2)

Q) If $a + \frac{1}{a} + 2 = 0$, then the value of $a^{37} - \frac{1}{a^{100}}$ is.

यदि $a + \frac{1}{a} + 2 = 0$, तो $a^{37} - \frac{1}{a^{100}}$ का मान है।

$a + \frac{1}{a} = -2$

$a = -1$

$(-1)^{37} - \frac{1}{(-1)^{100}}$

$-1 - \frac{1}{+1}$

$-1 - 1$

$= -2$

Q) If $a + \frac{1}{a} = -2$, then the value of $a^{1000} + a^{-1000}$ is.

यदि $a + \frac{1}{a} = -2$, तो $a^{1000} + a^{-1000}$ का मान है।

$a = -1$

$a^{1000} + \frac{1}{a^{1000}}$

$(-1)^{1000} + \frac{1}{(-1)^{1000}}$

$1 + 1$

$= 2$

Q) If $x + \frac{1}{x} = -1$, then the value of $x^5 + \frac{1}{x^5}$ will be:

यदि $x + \frac{1}{x} = -1$ है, तो $x^{15} + \frac{1}{x^{15}}$ का मान क्या होगा:

$$x^3 = +1$$

$$\begin{aligned} & \downarrow \\ & (x^3)^5 + \frac{1}{(x^3)^5} \\ & (+1)^5 + \frac{1}{(+1)^5} \\ & 1+1 \\ & = 2 \end{aligned}$$

• $x^{16} + \frac{1}{x^{16}} = ?$

$$\begin{aligned} & (x^3)^5 \times x + \frac{1}{(x^3)^5 \times x} \\ & 1 \times x + \frac{1}{1 \times x} \\ & x + \frac{1}{x} \\ & = -1 \end{aligned}$$

• $x^{17} + \frac{1}{x^{17}} = ?$

$$\begin{aligned} & (x^3)^5 \times x^2 + \frac{1}{(x^3)^5 \times x^2} \\ & 1 \times x^2 + \frac{1}{1 \times x^2} \\ & x^2 + \frac{1}{x^2} \\ & (-1)^2 - 2 = 1 - 2 = -1 \end{aligned}$$

Q) If $x + \frac{1}{x} = \sqrt{3}$, then the value of $x^{18} + x^{12} + x^6 + 1$ is.

यदि $x + \frac{1}{x} = \sqrt{3}$, तो $x^{18} + x^{12} + x^6 + 1$ का मान क्या होगा?

$$x^6 = -1$$

$$\begin{aligned} & \downarrow \\ & (x^6)^3 + (x^6)^2 + x^6 + 1 \\ & (-1)^3 + (-1)^2 + (-1) + 1 \\ & -1 + 1 - 1 + 1 \\ & = 0 \end{aligned}$$

Q) If $a^3 + \frac{1}{a^3} = 2$, then the value of $\frac{a^2+1}{a}$ is ? (where a is a positive number)

यदि $a^3 + \frac{1}{a^3} = 2$ हो, तो $\frac{a^2+1}{a}$ का मान कितना है? (जहाँ a एक धनात्मक संख्या है)

$$a = 1$$

$$a + \frac{1}{a} = 2$$

TYPE-VI

Q) If $3^a = 27^b = 81^c$ and $abc = 144$, then the value of $12\left(\frac{1}{a} + \frac{1}{2b} + \frac{1}{3c}\right)$ is.

यदि $3^a = 27^b = 81^c$ और $abc = 144$ है, तब $12\left(\frac{1}{a} + \frac{1}{2b} + \frac{1}{3c}\right)$ का मान है ?

$$3^a = 3^{3b} = 3^{4c}$$

$$a = 3b = 4c$$

$$\begin{array}{ccc} a & : & b & : & c \\ 12 & : & 4 & : & 3 \\ 12R & & 4R & & 3R \\ \hline 12 & & 4 & & 3 \end{array}$$

$$144R^3 = 144$$

$$R^3 = 1$$

$$(R=1)$$

$$12 \left[\frac{1}{12} + \frac{1}{8} + \frac{1}{15} \right]$$

$$12 \left(\frac{10+15+8}{120} \right)$$

$$= \frac{33}{10}$$

Q) If $x^{2a} = y^{2b} = z^{2c} \neq 0$ and $x^2 = yz$, then the value of $\frac{ab+bc+ca}{bc}$ is

यदि $x^{2a} = y^{2b} = z^{2c} \neq 0$ और $x^2 = yz$, तब $\frac{ab+bc+ca}{bc}$ है।

$$x^{2a} = y^{2b} = z^{2c} = k$$

$$x^{2a} = k \quad y^{2b} = k \quad z^{2c} = k$$

$$\boxed{x = k^{\frac{1}{2a}}} \quad \boxed{y = k^{\frac{1}{2b}}} \quad \boxed{z = k^{\frac{1}{2c}}}$$

$$\left(k^{\frac{1}{2a}} \right)^2 = k^{\frac{1}{2b}} \times k^{\frac{1}{2c}}$$

$$k^{\frac{1}{a}} = k^{\frac{1}{2b} + \frac{1}{2c}}$$

$$\frac{1}{a} = \frac{1}{2} \left[\frac{1}{b} + \frac{1}{c} \right]$$

$$\frac{2}{a} = \frac{b+c}{bc}$$

$$1 + \frac{ab+ac}{bc} = 2+1$$

$$\frac{bc+ab+ac}{bc} = 3$$

Q) If $2^{x+2y} = 256$ and $3^{x-2y} = \frac{1}{27}$, then the value of $x+y$ is.

यदि $2^{x+2y} = 256$ और $3^{x-2y} = \frac{1}{27}$, तो $x+y$ का मान है।

$$2^{x+2y} = 2^8$$

$$x+2y=8$$

$$x = \frac{8+(-3)}{2} = \frac{5}{2}$$

$$3^{x-2y} = 3^{-3}$$

$$x-2y=-3$$

$$2y = \frac{8-(-3)}{2} = \frac{11}{2}$$

$$\boxed{y = \frac{11}{4}}$$

$$\frac{5}{2} + \frac{11}{4}$$

$$\frac{20+22}{8}$$

$$\frac{42}{8} = \frac{21}{4}$$

Q) If $A = 1 + 2^p$ and $B = 1 + 2^{-p}$, then what is the value of B ?

यदि $A = 1 + 2^p$ तथा $B = 1 + 2^{-p}$ है, तो B का मान क्या है।

$$\boxed{2^p = A-1}$$

$$\boxed{B = 1 + \frac{1}{2^p}}$$

$$B = 1 + \frac{1}{A-1}$$

$$B = \frac{A - \cancel{x} + \cancel{x}}{A-1}$$

$$B = \frac{A}{A-1}$$

TYPE-VII

Q) If $\frac{8+2\sqrt{3}}{3\sqrt{3}+5} = a\sqrt{3} - b$, then find the value of $a+b$.

यदि $\frac{8+2\sqrt{3}}{3\sqrt{3}+5} = a\sqrt{3} - b$ है, तो $a+b$ का मान ज्ञात कीजिए।

$$\frac{(8+2\sqrt{3}) \times (3\sqrt{3}-5)}{(3\sqrt{3}+5)(3\sqrt{3}-5)}$$

$$\frac{24\sqrt{3} - 40 + 18 - 10\sqrt{3}}{27 - 25}$$

$$\frac{14\sqrt{3} - 22}{2} = \frac{2(7\sqrt{3} - 11)}{2}$$

$$7\sqrt{3} - 11 = a\sqrt{3} - b$$

On Comparing

$$a = 7$$

$$b = 11$$

$$a+b = 11+7 = 18$$

Q) If $\frac{\sqrt{38-5\sqrt{3}}}{\sqrt{26+7\sqrt{3}}} = \frac{a+b\sqrt{3}}{23}$, $b > 0$, then the value of $(b-a)$ is ?

यदि $\frac{\sqrt{38-5\sqrt{3}}}{\sqrt{26+7\sqrt{3}}} = \frac{a+b\sqrt{3}}{23}$, $b > 0$ हो, तो $(b-a)$ का मान कितना होगा ?

$$\frac{\sqrt{(38-5\sqrt{3}) \times \frac{2}{2}}}{\sqrt{(26+7\sqrt{3}) \times \frac{2}{2}}} = \frac{\sqrt{76-2 \times 5\sqrt{3}}}{\sqrt{52+2 \times 7\sqrt{3}}} = \frac{\sqrt{76-2 \times 5\sqrt{3}}}{\sqrt{52+2 \times 7\sqrt{3}}} < \frac{5\sqrt{3}}{7}$$

$$\frac{5\sqrt{3}-1}{7+\sqrt{3}} \cdot \frac{(5\sqrt{3}-1)}{7+\sqrt{3}} \times \frac{(7-\sqrt{3})}{7-\sqrt{3}}$$

$$\frac{35\sqrt{3} - 15 - 7 + \sqrt{3}}{49 - 3}$$

$$\frac{36\sqrt{3} - 22}{46}$$

$$\frac{2(18\sqrt{3} - 11)}{46}$$

$$\frac{18\sqrt{3} - 11}{23} = \frac{a + b\sqrt{3}}{23}$$

$$a = -11$$

$$b = 18$$

$$b - a$$

$$18 - (-11)$$

$$= 18 + 11 = 29$$

Q) If $x = 97 + 56\sqrt{3}$, then what is the value of $\sqrt[4]{x} + \frac{1}{\sqrt[4]{x}}$?

यदि $x = 97 + 56\sqrt{3}$ है, तो $\sqrt[4]{x} + \frac{1}{\sqrt[4]{x}}$ का मान क्या है ?

$$\sqrt{x} = \sqrt{97 + 2 \times 28\sqrt{3}}$$

$$7 \quad 4\sqrt{3}$$

$$2\sqrt{x} = 7 + 4\sqrt{3}$$

$$\sqrt[4]{x} = \sqrt{7 + 4\sqrt{3}}$$

$$2 \times 2\sqrt{3}$$

$$2 \quad \sqrt{3}$$

$$\boxed{\sqrt[4]{x} = 2 + \sqrt{3}}$$

$$\boxed{\frac{1}{\sqrt[4]{x}} = 2 - \sqrt{3}}$$

$$2 + \sqrt{3} + 2 - \sqrt{3} = 4$$

Q) If $x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$ and $z = 1$, then what is the value of

$$\left(\frac{x}{yz}\right) + \left(\frac{y}{xz}\right) + \left(\frac{z}{xy}\right) + 2\left[\left(\frac{1}{x}\right) + \left(\frac{1}{y}\right) + \left(\frac{1}{z}\right)\right] ?$$

यदि $x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$ तथा $z = 1$ है, तो $\left(\frac{x}{yz}\right) + \left(\frac{y}{xz}\right) + \left(\frac{z}{xy}\right) + 2\left[\left(\frac{1}{x}\right) + \left(\frac{1}{y}\right) + \left(\frac{1}{z}\right)\right]$ का मान क्या है ?

$$\frac{x^2}{xyz} + \frac{y^2}{xyz} + \frac{z^2}{xyz} + 2 \left[\frac{yz + xz + xy}{xyz} \right]$$

$$\frac{1}{xyz} [x^2 + y^2 + z^2 + 2(xy + yz + zx)]$$

$$\frac{(x+y+z)^2}{xyz} = \frac{(2+\sqrt{3}+2-\sqrt{3}+1)^2}{(2+\sqrt{3})(2-\sqrt{3}) \times 1} = \frac{5^2}{1 \times 1} = 25$$

Q) If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then what is the value of $x^3 - y^3$?

यदि $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ और $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, तो $x^3 - y^3$ का मान क्या है?

$$xy = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\boxed{y = \frac{1}{x}}$$

$$\hookrightarrow x^3 - \frac{1}{x^3}$$

$$x = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\frac{(\sqrt{3}+1)^2}{3-1} = \frac{3+1+2\sqrt{3}}{2}$$

$$\frac{4+2\sqrt{3}}{2} \Rightarrow \frac{2(2+\sqrt{3})}{2}$$

$$x = 2+\sqrt{3}$$

$$\frac{1}{x} = 2-\sqrt{3}$$

$$x - \frac{1}{x} = 2+\sqrt{3} - 2+\sqrt{3}$$

$$\boxed{2\sqrt{3}}$$

$$x^3 - \frac{1}{x^3} = (2\sqrt{3})^3 + 3 \times 2\sqrt{3}$$

$$24\sqrt{3} + 6\sqrt{3}$$

$$30\sqrt{3}$$

Q) If $(2+\sqrt{3})a = (2-\sqrt{3})b = 1$, then the value of $\frac{1}{a} + \frac{1}{b}$ is?

यदि $(2+\sqrt{3})a = (2-\sqrt{3})b = 1$, तो $\frac{1}{a} + \frac{1}{b}$ का मान क्या है?

$$2 + \sqrt{3} = \frac{1}{a} \quad 2 - \sqrt{3} = \frac{1}{b}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{2 + \sqrt{3} + 2 - \sqrt{3}}{4}$$

TYPE-VIII

- $16 - x^2$

for Max. value put $x^2 = 0$

$$16 - 0 \\ = 16$$

Ex:- • $2x^2 + 4x + 1$

↓
Min. value =

∴ Differentiation:-

- $\frac{d}{dx} x^5 = 5x^{5-1} = 5x^4$

- $\frac{d}{dx} x^4 = 4x^{4-1} = 4x^3$

- $\frac{d}{dx} x^3 = 3x^{3-1} = 3x^2$

- $\frac{d}{dx} x^2 = 2x^{2-1} = 2x$

- $5x$

$$5x^1$$

$$5 \times x^{1-1} \Rightarrow 5 \times 1 = 5$$

- 6

$$\text{diff} = 0$$

★ x^n differentiation

$$= nx^{n-1}$$

Ex:- • $8x^2 + 24x + 19$
min value = ?

$$16x + 24 + 0 = 0$$

$$16x + 24 = 0$$

$$16x = -24$$

$$x = \frac{-24}{16} - \frac{3}{2}$$

- $16 + x^2$

for minimum value

$$x^2 = 0$$

$$16 + 0$$

$$= 16$$

$$4x^1 + 4 + 0 = 0$$

$$4x + 4 = 0$$

$$4x = -4$$

$$x = -1$$

Min value:- $2(-1)^2 + 4(-1) + 1$

$$2 - 4 + 1$$

$$3 - 4$$

$$= -1$$

$$\begin{aligned}
 & 8 \times \left(\frac{-3}{2}\right)^2 + 24 \left(\frac{-3}{2}\right) + 19 \\
 & \cancel{2} \times \frac{9}{4} - \cancel{24} \times \frac{3}{2} + 19 \\
 & 18 - 36 + 19 \\
 & 37 - 36 \\
 & = (+1)
 \end{aligned}$$

Q) What will be the minimum value of $(4x^2 + 2x + 1)$?
 $(4x^2 + 2x + 1)$ का न्यूनतम मान क्या होगा?

$$\begin{aligned}
 8x + 2 + 0 &= 0 \\
 8x &= -2 \\
 \boxed{x = \frac{-2}{8} = \frac{-1}{4}} \\
 \text{Min. value} &= 4 \times \left(\frac{-1}{4}\right)^2 + 2 \left(\frac{-1}{4}\right) + 1 \\
 &= 4 \times \frac{1}{16} - 2 \times \frac{1}{4} + 1 \\
 &= \frac{1}{4} - \frac{2}{4} + 1 \\
 &= \frac{1-2}{4} + 1 \\
 &= 1 - \frac{1}{4} \\
 &= \frac{3}{4}
 \end{aligned}$$

Q) If x is real, then what will be maximum value of $(7 + 10x - 5x^2)$?
 यदि x वास्तविक हो, तो $7 + 10x - 5x^2$ का अधिकतम मान क्या होगा?

$$\begin{aligned}
 0 + 10 - 10x &= 0 \\
 10x &= 10 \\
 \boxed{x = 1} \\
 \text{Max. Value} &= 7 + 10 \times 1 - 5 \times (1)^2 \\
 &= 17 - 5 = 12
 \end{aligned}$$

$a \times b \rightarrow \text{Max. Value}$

↓
Max. Value $\Rightarrow \boxed{a=b}$

where
 $(a+b) = \text{Constant}$

Q) If $x+y+z=13$, then find the maximum value of $(x-2)(y+1)(z-3)$.

यदि $x+y+z=13$ हो, तो $(x-2)(y+1)(z-3)$ का अधिकतम मान बताये।

$$\overset{-2}{x} + \overset{+1}{y} + \overset{-3}{z} = 13 - 2 + 1 - 3$$

$$\underbrace{(x-2)}_3 + \underbrace{(y+1)}_3 + \underbrace{(z-3)}_3 = 9$$

$$\begin{matrix} \rightarrow \\ 3 \times 3 \times 3 \\ 27 \end{matrix}$$

Ex:- xy की Max. Value

$$\downarrow \\ 3 \times 3$$

⑨

जहाँ $x+y=6$
 $\downarrow \quad \downarrow$
 $3 \quad 3$

Max. Value at $x=y$

Ex:- xyz की Max. Value

$$\downarrow \\ 6 \times 6 \times 6 \\ 216$$

Max. value at
 $x=y=z$

where
 $x+y+z=18$
 $\downarrow \quad \downarrow \quad \downarrow$
 $6 \quad 6 \quad 6$

$a+b \rightarrow \text{Min. Value}$

↓
Minimum value at
 $a=b$

where
 $a \times b = \text{Constant}$

Ex:- • $a \times b = 7$

$(a+b) \Rightarrow \text{Min. value} = ?$

Min. at $\Rightarrow a=b$

↓
 $a \times b = 7$

$$a^2 = 7$$

$$a = \sqrt{7} = b$$

$$\sqrt{7} + \sqrt{7}$$

$$2\sqrt{7}$$

Ex:- $a \times b = 49$

$a + b \Rightarrow \text{Min. Value} = ?$

$a \times b = 49$

$\downarrow \quad \downarrow$
 $7 \quad 7$

$a + b \rightarrow \text{Min Value} \rightarrow 7 + 7$
 14

Q) If $(a-7)(b-10)(c-12) = 1000$, then what will be the minimum value of $(a+b+c)$?

यदि $(a-7)(b-10)(c-12) = 1000$ हो, तो $(a+b+c)$ का न्यूनतम मान क्या होगा?

$(a-7)(b-10)(c-12) = 1000$

$a-7=10 \quad b-10=10 \quad c-12=10$

$a=17$

$b=20$

$c=22$

$\downarrow$
 $17+20+22$
 59

Q) If $3x+5y+7z=49$ and $9x+8y+21z=126$ then what is the value of y ?

यदि $3x+5y+7z=49$ तथा $9x+8y+21z=126$ है, तो y का मान क्या है?

$3x+5y+7z=49$ — ①

$9x+8y+21z=126$ — ②

eq ① $\times$ ③ $\Rightarrow 9x+15y+21z=49 \times 3=147$ — ③

③ - ②

$15y-8y=147-126$

$7y=21$

$y=3$

Q) If $2x+3y-5z=18$, $3x+2y+z=29$ and $x+y+3z=17$, then what is the value of $xy+yz+zx$?

यदि $2x+3y-5z=18$, $3x+2y+z=29$ तथा $x+y+3z=17$ है, तो $xy+yz+zx$ का मान क्या है?

$$2x + 3y - 5z = 18 \quad \text{--- (1)}$$

$$3x + 2y + z = 29 \quad \text{--- (2)}$$

$$x + y + 3z = 17 \quad \text{--- (3)}$$

$$6x + 6y - z = 64 \quad \text{--- (4)}$$

$$\text{eq (3)} \times (6) \Rightarrow 6x + 6y + 18z = 102 \quad \text{--- (5)}$$

$$(5) - (4)$$

$$19z = 38$$

$$z = \frac{38}{19} = 2$$

①

$$3(2x + 3y = 18 + 10 = 28)$$

$$2(3x + 2y = 29 - 2 = 27)$$

$$6x + 9y = 84$$

$$6x + 4y = 54$$

$$5y = 30$$

$$y = 6$$

$$x + 6 + 6 = 17$$

$$x = 17 - 12 = 5$$

$$5 \times 6 + 6 \times 2 + 2 \times 5$$

$$30 + 12 + 10$$

$$52$$

MISCELLANEOUS

Q) निम्नलिखित व्यंजक को सरल कीजिए।

Simplify the following expression

$$(1+x)(1+x^2)(1+x^4)(1+x^8)(1-x)$$

$$(1-x^2)(1+x^2)(1+x^4)(1+x^8) \rightarrow (1-x^{16})$$

$$(1-x^4)(1+x^4)(1+x^8)$$

$$(1-x^8)(1+x^8)$$

$$(1-x^{16})$$

Q) निम्नलिखित व्यंजक को सरल कीजिए।

Simplify the following expression.

$$(x+y)(x-y)(x^2+y^2)(x^4+y^4)(x^8+y^8)(x^{16}+y^{16}).$$

$$(x^{32} - y^{32})$$

Q) दिए गए व्यंजक को सरल कीजिए।

Simplify the given expression:

$$\left(k - \frac{1}{k}\right) \left(k^2 + \frac{1}{k^2}\right) \left(k^4 + \frac{1}{k^4}\right) \left(k^8 + \frac{1}{k^8}\right) \left(k^{16} + \frac{1}{k^{16}}\right) \left(k^{32} + \frac{1}{k^{32}}\right)$$

$$\frac{k + \frac{1}{k}}{k + \frac{1}{k}}$$

$$\frac{k^{64} - \frac{1}{k^{64}}}{k + \frac{1}{k}}$$

Q) $p^3 + q^3 + r^3 - 3pqr = 4$, if $a = q + r$, $b = r + p$ and $c = p + q$, then what is the value of $a^3 + b^3 + c^3 - 3abc$?

$p^3 + q^3 + r^3 - 3pqr = 4$ है, यदि $a = q + r$, $b = r + p$ तथा $c = p + q$ है, तो $a^3 + b^3 + c^3 - 3abc$ का मान क्या है?

$$\boxed{p^3 = 4}$$

$$p^3 + p^3 = 0$$

$$2p^3$$

$$2 \times 4 = 8$$

$$\begin{aligned} a &= 0 \\ b &= p \\ c &= p \end{aligned}$$

Q) If $a + b + c = 0$, then the value of $\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)}$?

यदि $a + b + c = 0$ हो, तो $\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)}$ मान निकालें

$$\frac{(c+a)}{(a+b)(b+c)(c+a)} + \frac{a+b}{(a+b)(b+c)(c+a)} + \frac{b+c}{(a+b)(b+c)(c+a)}$$

$$\frac{c+a+a+b+b+c}{(a+b)(b+c)(c+a)}$$

$$\frac{2(a+b+c)}{(a+b)(b+c)(c+a)} = 0$$

Q) का मान ज्ञात कीजिए / The value of

$$\frac{p^2 - (q-r)^2}{(p+r)^2 - q^2} + \frac{q^2 - (p-r)^2}{(p+q)^2 - r^2} + \frac{r^2 - (p-q)^2}{(q+r)^2 - p^2}$$

$$\frac{(p+q-r)(p-q+r)}{(p+q+r)(p-r-q)} + \frac{(q+p-r)(q-p+r)}{(p+q-r)(p+q+r)} + \frac{(r+p-q)(r-p+q)}{(q+r-p)(q+r+p)}$$

$$\frac{p+q-r+q-p+r+r+p-q}{p+q+r} = \frac{p+q+r}{p+q+r} = 1$$

Q) If $x^2 + 4y^2 + 2x + 1 = 0$, then find the value of $x^{39} + y^{36}$
 यदि $x^2 + 4y^2 + 2x + 1 = 0$ है, तो $x^{39} + y^{36}$ का मान ज्ञात कीजिए।

$$\begin{aligned} x^2 + 2x + 1 + 4y^2 &= 0 \\ (x+1)^2 + 4y^2 &= 0 \\ \downarrow & \quad \downarrow \\ x+1=0 & \quad y=0 \\ \boxed{x=-1} & \end{aligned}$$

$$\begin{aligned} x^2 + 4y^2 + 1 &= -2x + 0y \\ 2[-x + 0y] & \\ x &= \frac{-1}{1} = -1 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} & \rightarrow (-1)^{39} + 0 \\ &= -1 \end{aligned}$$

Q) If $8(x+y)^3 - 27(x-y)^3 = (5y-x)(Ax^2 + By^2 + Cxy)$, then what is the value of $(A+B-C)$?

यदि $8(x+y)^3 - 27(x-y)^3 = (5y-x)(Ax^2 + By^2 + Cxy)$, तब $(A+B-C)$ का मान क्या है?

$$\begin{aligned} & [2(x+y)]^3 - [3(x-y)]^3 \\ (2x+2y-3x+3y) & [4(x+y)^2 + 9(x-y)^2 + 2(x+y) \times 3(x-y)] \\ (5y-x) & [4(x^2+y^2+2xy) + 9(x^2+y^2-2xy) + 6(x^2-y^2)] \\ & (5y-x)(19x^2+7y^2-10xy) \end{aligned}$$

$$A = 19$$

$$B = 7$$

$$C = -10$$

$$A + B - C$$

$$19 + 7 - (-10)$$

$$19 + 7 + 10$$

$$= 36$$

1. If $x^2 + \frac{1}{x^2} = \sqrt{5}$, find $x^8 + \frac{1}{x^8} = ?$

यदि $x^2 + \frac{1}{x^2} = \sqrt{5}$, तो $x^8 + \frac{1}{x^8} =$ ज्ञात कीजिए।

- (a) 9
- (b) 7
- (c) 11
- (d) 8

2. If $\sqrt{x} - \frac{1}{\sqrt{x}} = 3$, find $x^2 + \frac{1}{x^2} = ?$

यदि $\sqrt{x} - \frac{1}{\sqrt{x}} = 3$, तो $x^2 + \frac{1}{x^2} =$ ज्ञात कीजिए।

- (a) 119
- (b) 121
- (c) 123
- (d) 120

3. If $x^2 + \frac{1}{x^2} = 322$ find $\sqrt{x} - \frac{1}{\sqrt{x}} = ?$

यदि $x^2 + \frac{1}{x^2} = 322$ तो $\sqrt{x} - \frac{1}{\sqrt{x}} = ?$

- (a) 4
- (b) 2
- (c) 20
- (d) 10

4. If $x + \frac{1}{x} = \sqrt{5}$, find $x^4 + \frac{1}{x^4} = ?$

यदि $x + \frac{1}{x} = \sqrt{5}$, तो $x^4 + \frac{1}{x^4} =$ ज्ञात कीजिए।

- (a) 6
- (b) 4
- (c) 7
- (d) 8

5. If $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{7}$ find $x^2 + \frac{1}{x^2} = ?$

यदि $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{7}$ तो $x^2 + \frac{1}{x^2} = ?$ ज्ञात कीजिये।

- (a) 20
- (b) 25
- (c) 21
- (d) 23

6. If $x = 7 - 4\sqrt{3}$, then find the value of $\left(x^2 + \frac{1}{x^2}\right)$

यदि $x = 7 - 4\sqrt{3}$ है, तो $\left(x^2 + \frac{1}{x^2}\right)$ का मान ज्ञात कीजिए।

- (a) 197
- (b) 195
- (c) 196
- (d) 194

7. If $x - \frac{1}{x} = 5$, $x \neq 0$, then what is the value of $\frac{x^6 + 3x^3 - 1}{x^6 - 8x^3 - 1}$?

यदि $x - \frac{1}{x} = 5$, $x \neq 0$ है, तो $\frac{x^6 + 3x^3 - 1}{x^6 - 8x^3 - 1}$ का मान ज्ञात कीजिए।

- (a) $\frac{3}{8}$
 (b) $\frac{13}{12}$
 (c) $\frac{4}{9}$
 (d) $\frac{11}{13}$

8. If $x + \frac{1}{x} = 2$, then the value of $x^{17} + \frac{1}{x^{17}}$ is.

यदि $x + \frac{1}{x} = 2$ है, तो $x^{17} + \frac{1}{x^{17}}$ का मान है।

- (a) 2^{17}
 (b) 4
 (d) 2
 (d) 8

9. If $a + \frac{1}{a} = 2$, then the value of $(a^{118} + \frac{1}{a^{117}})$ is.

यदि $a + \frac{1}{a} = 2$ है, तो $(a^{118} + \frac{1}{a^{117}})$ का मान है।

- (a) 117
 (b) 2
 (c) 118
 (d) 1

10. If $x + \frac{1}{x} = 2$, $x \neq 0$, then the value of $x^2 + \frac{1}{x^3}$ is.

यदि $x + \frac{1}{x} = 2$, $x \neq 0$ है, तो $x^2 + \frac{1}{x^3}$ का मान है।

- (a) 4
 (b) 2
 (c) 3
 (d) N.O.T.

11. If $3^a = 27^b = 81^c$ and $abc = 144$, then the value of $12 \left(\frac{1}{a} + \frac{1}{2b} + \frac{1}{5c} \right)$ is.

यदि $3^a = 27^b = 81^c$ और $abc = 144$ है, तब $12 \left(\frac{1}{a} + \frac{1}{2b} + \frac{1}{5c} \right)$ का मान है।

- (a) $\frac{18}{120}$
 (b) $\frac{18}{10}$
 (c) $\frac{33}{10}$
 (d) $\frac{17}{120}$

12. If $2^x = 4^y = 8^z$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{4z} = 4$, then find the value of x ?

यदि $2^x = 4^y = 8^z$ और $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{4z} = 4$ हो, तो x का मान ज्ञात कीजिए?

- (a) 7 (b) $\frac{1}{16}$
 (c) $\frac{7}{16}$ (d) N.O.T.

13. If $\sqrt{28 - 6\sqrt{3}} = \sqrt{3}a + b$, (where a, b are rational), value of $(a + b)$ is.

यदि $\sqrt{28 - 6\sqrt{3}} = \sqrt{3}a + b$, (जहाँ a तथा b परिमेय है) $(a + b)$ का मान है।

(a) 2

(b) -2

(c) -1

(d) 1

14. If $x = \sqrt{5} + \sqrt{3}$ and $y = \sqrt{5} - \sqrt{3}$, then the value of $x^4 - y^4$ is?

यदि $x = \sqrt{5} + \sqrt{3}$ और $y = \sqrt{5} - \sqrt{3}$, तो $x^4 - y^4$ का मान क्या होगा?

(a) 16

(b) 544

(c) $32\sqrt{15}$

(d) $64\sqrt{15}$

15. If $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\frac{x^2+xy+y^2}{x^2-xy+y^2}$

यदि $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ और $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, तो $\frac{x^2+xy+y^2}{x^2-xy+y^2}$ का मान है।

(a) $5/3$

(b) $4/3$

(c) $3/5$

(d) $3/4$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
B	A	A	C	D	D	B	C	B	B
11	12	13	14	15					
C	C	A	D	B					

Worksheet Solution

$$(1) \quad n^2 + \frac{1}{n^2} = \sqrt{5}$$

$$\Rightarrow \quad n^4 + \frac{1}{n^4} = 5 - 2$$

$$\Rightarrow \quad n^4 + \frac{1}{n^4} = 3$$

$$\Rightarrow \quad n^8 + \frac{1}{n^8} = 3^2 - 2$$

$$\Rightarrow \quad n^8 + \frac{1}{n^8} = 9 - 2$$

$$\Rightarrow \quad n^8 + \frac{1}{n^8} = 7 \text{ Ans}$$

$$(2) \quad \sqrt{n} - \frac{1}{\sqrt{n}} = 3$$

वर्ग करने पर

$$\Rightarrow \quad n + \frac{1}{n} = 3^2 + 2$$

$$\Rightarrow \quad n + \frac{1}{n} = 11$$

$$\Rightarrow \quad n^2 + \frac{1}{n^2} = 11^2 - 2$$

$$\Rightarrow \quad n^2 + \frac{1}{n^2} = 119 \text{ Ans}$$

$$(3) \quad n^2 + \frac{1}{n^2} = 322$$

$$\Rightarrow \quad n + \frac{1}{n} = \sqrt{322 + 2}$$

$$\Rightarrow \quad n + \frac{1}{n} = 18$$

$$\Rightarrow \quad \sqrt{n} - \frac{1}{\sqrt{n}} = \sqrt{18 - 2}$$

$$\Rightarrow \quad \sqrt{n} - \frac{1}{\sqrt{n}} = 4 \text{ Ans}$$

$$(4) \quad n + \frac{1}{n} = \sqrt{5}$$

$$n^2 + \frac{1}{n^2} = (\sqrt{5})^2 - 2$$

$$n^2 + \frac{1}{n^2} = 3$$

$$n^4 + \frac{1}{n^4} = 3^2 - 2$$

$$n^4 + \frac{1}{n^4} = 9 - 2$$

$$n^4 + \frac{1}{n^4} = 7 \text{ Ans}$$

$$(5) \quad \sqrt{n} + \frac{1}{\sqrt{n}} = \sqrt{7}$$

$$n + \frac{1}{n} = (\sqrt{7})^2 - 2$$

$$n + \frac{1}{n} = 5$$

$$n^2 + \frac{1}{n^2} = 5^2 - 2$$

$$n^2 + \frac{1}{n^2} = 23 \text{ Ans}$$

$$\textcircled{6} \quad n = 7 - 4\sqrt{3}$$

$$\frac{1}{n} = 7 + 4\sqrt{3}$$

~~$$\frac{n+1}{n} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$$~~

$$\frac{n+1}{n} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$$

$$\frac{n+1}{n} = 14$$

$$\frac{n^2+1}{n^2} = 14^2 - 2$$

$$\frac{n^2+1}{n^2} = 194 \text{ Ans}$$

$$\textcircled{7} \quad n - \frac{1}{n} = 5$$

$$\frac{n^6 + 3n^3 - 1}{n^6 - 8n^3 - 1} = \frac{\left(n^3 - \frac{1}{n^3}\right) + 3}{\left(n^3 - \frac{1}{n^3}\right) - 8}$$

अंश व हर को n^3 से भाग देने पर

$$\Rightarrow \frac{(5)^3 + 3 \times 5 + 3}{(5)^3 + 3 \times 5 - 8}$$

$$\Rightarrow \frac{125 + 15 + 3}{125 + 15 - 8}$$

$$\Rightarrow \frac{143}{132} = \frac{13}{12} \text{ Ans}$$

$$\textcircled{8} \quad n + \frac{1}{n} = 2$$

$$n = 1$$

$$\frac{n^{17} + 1}{n^{17}} = \frac{(1)^{17} + 1}{(1)^{17}}$$

$$\frac{1+1}{1} = 2 \text{ Ans}$$

$$\textcircled{9} \quad a + \frac{1}{a} = 2$$

$$\Rightarrow a = 1$$

$$\Rightarrow \frac{a^{118} + 1}{a^{118}} = \frac{(1)^{118} + 1}{(1)^{118}}$$

$$\Rightarrow \frac{1+1}{1} = 2 \text{ Ans}$$

$$\textcircled{10} \quad n + \frac{1}{n} = 2$$

$$n = 1$$

$$\frac{n^2 + 1}{n^3} = \frac{(1)^2 + 1}{(1)^3}$$

$$\frac{1+1}{1} = 2 \text{ Ans}$$

$$\textcircled{11} \quad 3^a = 27^b = 81^c$$

$$3^a = 3^{3b} = 3^{4c}$$

$a = 12, b = 4, c = 3$ मान रखने पर

$$12\left(\frac{1}{a} + \frac{1}{2b} + \frac{1}{5c}\right) = 12\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{15}\right)$$

$$\Rightarrow 12\left(\frac{10 + 15 + 8}{120}\right)$$

$$12 \times \frac{33}{120} = \frac{33}{10} \text{ Ans}$$

$$(12) \quad 2^n = 4^y = 8^z$$

$$2^n = 2^{2y} = 2^{3z}$$

$$n = 2y = 3z$$

$$y = \frac{n}{2}, z = \frac{n}{3}$$

$$\Rightarrow \frac{1}{2n} + \frac{1}{4y} + \frac{1}{4z} = 4$$

$$\Rightarrow \frac{1}{2n} + \frac{1}{4(n/2)} + \frac{1}{4(n/3)} = 4$$

$$\Rightarrow \frac{1}{2n} + \frac{1}{2n} + \frac{3}{4n} = 4$$

$$\Rightarrow \frac{2+2+3}{4n} = 4$$

$$\Rightarrow \frac{7}{4n} = 4$$

$$\Rightarrow n = \frac{7}{16} \text{ Ans}$$

$$(13) \quad \sqrt{20-6\sqrt{3}} = \sqrt{3a+b}$$

$$\sqrt{27+1-2\sqrt{3}\sqrt{3}\times} = \sqrt{3a+b}$$

$$\sqrt{(3\sqrt{3}-1)^2} = \sqrt{3a+b}$$

$$3\sqrt{3}-1 = \sqrt{3a+b}$$

Compare करने पर

$$a=3, b=-1$$

$$(a+b) = 3+(-1)$$

$$a+b = \underline{2 \text{ Ans}}$$

$$(14) \quad x = \sqrt{5} + \sqrt{3}$$

$$x^2 = 8 + 2\sqrt{15}$$

$$x^4 = 124 + 32\sqrt{15}$$

और

$$y = \sqrt{5} - \sqrt{3}$$

$$y^2 = 8 - 2\sqrt{15}$$

$$y^4 = 124 - 32\sqrt{15}$$

$$x^4 - y^4 = 124 + 32\sqrt{15} - 124 + 32\sqrt{15}$$

$$\Rightarrow \underline{64\sqrt{15} \text{ Ans}}$$

$$(15) \quad x = \frac{\sqrt{5}+1}{\sqrt{5}-1}, y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$$

$$x+y = \frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1}$$

$$\frac{5+1+2\sqrt{5}+5+1-2\sqrt{5}}{5-1}$$

$$\Rightarrow 3$$

इसी प्रकार:

$$x-y = \frac{4\sqrt{5}}{4}$$

$$x-y=\sqrt{5}$$

उदा

$$xy=1$$

$$\frac{x^2+xy+y^2}{x^2-xy+y^2} = \frac{(x+y)^2-xy}{(x-y)^2+xy}$$

$$\frac{9-1}{5+1} = \frac{8}{6} = \frac{4}{3} \text{ Ans}$$