

ALGEBRA

Q) If $(x^2 - \frac{1}{x^2}) = 4\sqrt{6}$ and $x > 1$, what is the value of $(x^3 - \frac{1}{x^3})$?

यदि $(x^2 - \frac{1}{x^2}) = 4\sqrt{6}$ और $x > 1$, $(x^3 - \frac{1}{x^3})$ का मान क्या है ?

$$\downarrow$$

$$x^4 + \frac{1}{x^4} = (4\sqrt{6})^2 + 2$$

$$96 + 2 = 98$$

$$x^2 + \frac{1}{x^2} = \sqrt{98 + 2} = 10$$

$$\downarrow$$

$$x - \frac{1}{x} = \sqrt{10 - 2} = \sqrt{8}$$

$$\downarrow k^3 + 3k$$

$$(4\sqrt{6})^3 + 3(4\sqrt{6})$$

$$8\sqrt{6} + 3\sqrt{6}$$

$$11\sqrt{6}$$

$$4 \times 2$$

$$11 \times 2\sqrt{2}$$

$$22\sqrt{2}$$

$$(\sqrt{a})^3 = a\sqrt{a}$$

$$\boxed{x + \frac{1}{x} = K}$$

$$\boxed{x^2 + \frac{1}{x^2} = K^2 - 2}$$

$$\boxed{x^2 + \frac{1}{x^2} = P}$$

$$\boxed{x^4 + \frac{1}{x^4} = P^2 - 2}$$

$$x^4 + \frac{1}{x^4} = Q$$

$$x^8 + \frac{1}{x^8} = Q^2 - 2$$

$$\bullet \quad x^4 - \frac{1}{x^4} = \left(x^2 - \frac{1}{x^2}\right) \left(x^2 + \frac{1}{x^2}\right)$$

$$\left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2}\right)$$

Q) If $a + \frac{1}{a} = 3$, then the value of $a^4 + \frac{1}{a^4}$ is.

यदि $a + \frac{1}{a} = 3$, तब $a^4 + \frac{1}{a^4}$ का मान है।

$$a^2 + \frac{1}{a^2} = 3^2 - 2 = 7$$

$$a^4 + \frac{1}{a^4} = 7^2 - 2$$

$$47$$

Q) If $(x - \frac{1}{x}) = 10$, what is the value of $(x^4 - \frac{1}{x^4})$?

यदि $(x - \frac{1}{x}) = 10$, $(x^4 - \frac{1}{x^4})$ का मान क्या है?

$$\hookrightarrow (x - \frac{1}{x})(x + \frac{1}{x})(x^2 + \frac{1}{x^2})$$

$$x + \frac{1}{x} = \sqrt{10^2 + 4}$$

$$= \sqrt{10^2 + 4}$$

$$= \sqrt{104}$$

$$x - \frac{1}{x} = 10$$

$$x^2 + \frac{1}{x^2} = 10^2 + 2$$

$$= 102$$

$$10 \times 102 \times \sqrt{104}$$

$$1020 \sqrt{104}$$

Q) If $x > 1$ and $x^2 + \frac{1}{x^2} = 2\sqrt{5}$, what is the value of $(x^4 - \frac{1}{x^4})$?

यदि $x > 1$ और $x^2 + \frac{1}{x^2} = 2\sqrt{5}$, $(x^4 - \frac{1}{x^4})$ का मान क्या है?

$$\hookrightarrow (x - \frac{1}{x})(x + \frac{1}{x})(x^2 + \frac{1}{x^2})$$

$$x - \frac{1}{x} = \sqrt{2\sqrt{5} - 2}$$

$$x + \frac{1}{x} = \sqrt{2\sqrt{5} + 2}$$

$$(\sqrt{2\sqrt{5} - 2})(\sqrt{2\sqrt{5} + 2})(2\sqrt{5})$$

$$\sqrt{(2\sqrt{5} - 2)(2\sqrt{5} + 2)}(2\sqrt{5})$$

$$\sqrt{20 - 4} \times 2\sqrt{5}$$

$$4 \times 2\sqrt{5} = 8\sqrt{5}$$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$(a+b)(a-b) = a^2 - b^2$$

Q) If $x^2 - \frac{1}{x^2} = 4\sqrt{2}$, what is the value of $x^4 - \frac{1}{x^4}$?

यदि $x^2 - \frac{1}{x^2} = 4\sqrt{2}$ है, $x^4 - \frac{1}{x^4}$ का मान क्या है?

$$\hookrightarrow x^4 + \frac{1}{x^4} = 32 + 2$$

$$34$$

↓

$$x^2 + \frac{1}{x^2} = \sqrt{34 + 2}$$

$$= 6$$

$$\hookrightarrow (x - \frac{1}{x})(x + \frac{1}{x})(x^2 + \frac{1}{x^2})$$

$$(x^2 - \frac{1}{x^2})(x^2 + \frac{1}{x^2})$$

$$4\sqrt{2} \times 6$$

$$24\sqrt{2}$$

Q) If $\sqrt{x} - \frac{1}{\sqrt{x}} = 4$, then $x^2 + \frac{1}{x^2}$ is equal to:

यदि $\sqrt{x} - \frac{1}{\sqrt{x}} = 4$, तो $x^2 + \frac{1}{x^2}$ बराबर है:

$$(\sqrt{x})^2 + \frac{1}{(\sqrt{x})^2} = 4^2 + 2$$

$$x + \frac{1}{x} = 18$$

$$x^2 + \frac{1}{x^2} = 18^2 - 2$$

$$322$$

Q) If $x - 3 = \frac{1}{2x}$, then find the value of $(x^4 + \frac{1}{16x^4})$ is.

यदि $x - 3 = \frac{1}{2x}$ है, तो $(x^4 + \frac{1}{16x^4})$ का मान ज्ञात कीजिए।

$$x - \frac{1}{2x} = 3$$

$$x^2 + \frac{1}{4x^2} = 3^2 + 2 \times 3 \times \frac{1}{2}$$

$$= 10$$

$$x^4 + \frac{1}{16x^4} = 10^2 - 2 \times 10 \times \frac{1}{4}$$

$$100 - \frac{1}{2} = 99\frac{1}{2}$$

Q) If $x^4 + \frac{1}{x^4} = 1154$ ($x > 0$), then the value of $(x + \frac{1}{x})$ is.

यदि $x^4 + \frac{1}{x^4} = 1154$ ($x > 0$) है, तो $(x + \frac{1}{x})$ का मान ज्ञात करें।

$$x^2 + \frac{1}{x^2} = \sqrt{1154 + 2}$$

$$\begin{array}{|l} \sqrt{1156} \\ = 34 \end{array}$$

$$x + \frac{1}{x} = \sqrt{34 + 2} = \sqrt{36} = 6$$

Q) If $k^4 + \frac{1}{k^4} = 194$, then find the value of $k^3 + \frac{1}{k^3}$ is

यदि $k^4 + \frac{1}{k^4} = 194$ है, तो $k^3 + \frac{1}{k^3}$ का मान ज्ञात कीजिए।

$$\begin{array}{l}
 k^2 + \frac{1}{k^2} = \sqrt{194+2} \\
 \quad \quad \quad = 14 \\
 \downarrow \\
 k + \frac{1}{k} = \sqrt{14+2} \\
 \quad \quad \quad = 4
 \end{array}
 \qquad
 \begin{array}{l}
 \downarrow \\
 4^3 - 3 \times 4 \\
 64 - 12 \\
 52
 \end{array}$$

$$\bullet \left(x^2 + \frac{1}{x^2}\right) \left(x^3 + \frac{1}{x^3}\right)$$

$$x^5 + \frac{1}{x} + x + \frac{1}{x^5}$$

$$x^5 + \frac{1}{x^5} + \left(x + \frac{1}{x}\right) - \left(x + \frac{1}{x}\right)$$

$$\boxed{x^5 + \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2}\right) \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)}$$

$$\bullet \left(x^2 + \frac{1}{x^2}\right) \left(x^3 - \frac{1}{x^3}\right)$$

$$x^5 - \frac{1}{x} + x - \frac{1}{x^5}$$

$$x^5 - \frac{1}{x^5} + \left(x - \frac{1}{x}\right) - \left(x - \frac{1}{x}\right)$$

$$= x^5 - \frac{1}{x^5}$$

$$\boxed{x^5 - \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2}\right) \left(x^3 - \frac{1}{x^3}\right) - \left(x - \frac{1}{x}\right)}$$

Q) If $\left(x + \frac{1}{x}\right) = 5$, then the value of $\left(x^5 + \frac{1}{x^5}\right)$ is

यदि $\left(x + \frac{1}{x}\right) = 5$ है, तब $\left(x^5 + \frac{1}{x^5}\right)$ का मान है।

$$\begin{array}{l}
 \downarrow \\
 x^2 + \frac{1}{x^2} = 5^2 - 2 = 23 \\
 \downarrow \\
 x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 \\
 \quad \quad \quad = 125 - 15 = 110
 \end{array}
 \qquad
 \begin{array}{l}
 \downarrow \\
 \left(x^2 + \frac{1}{x^2}\right) \left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right) \\
 \downarrow \\
 23 \times 110 - 5 \\
 2530 - 5 \\
 2525
 \end{array}$$

Q) If $a + \frac{1}{a} = 7$, then $a^5 + \frac{1}{a^5}$ is equal to?

यदि $a + \frac{1}{a} = 7$ है, तो $a^5 + \frac{1}{a^5}$ का मान किसके बराबर है?

$$\hookrightarrow \left(a^2 + \frac{1}{a^2}\right)\left(a^3 + \frac{1}{a^3}\right) - \left(a + \frac{1}{a}\right)$$

$$\hookrightarrow a^2 + \frac{1}{a^2} = 7^2 - 2 = 47$$

$$a^3 + \frac{1}{a^3} = 7^3 - 3 \times 7$$

$$343 - 21$$

$$= 322$$

$$47 \times 322 - 7$$

$$15134 - 7$$

$$15127$$

Q) If $x - \frac{1}{x} = -6$, then what will be the value of $x^5 - \frac{1}{x^5}$?

यदि $x - \frac{1}{x} = -6$ है, तो $x^5 - \frac{1}{x^5}$ का मान कितना होगा?

$$\hookrightarrow \left(x^2 + \frac{1}{x^2}\right)\left(x^3 - \frac{1}{x^3}\right) - \left(x - \frac{1}{x}\right)$$

$$\downarrow$$

$$x^2 + \frac{1}{x^2} = (-6)^2 + 2$$

$$= 38$$

$$x^3 - \frac{1}{x^3} = (-6)^3 + 3(-6)$$

$$-216 - 18$$

$$= -234$$

$$\downarrow$$

$$38 \times (-234) - (-6)$$

$$-8892 + 6$$

$$= -8886$$

Q) If $x^2 - \sqrt{7}x + 1 = 0$, then what is the value of $x^5 + \frac{1}{x^5}$?

यदि $x^2 - \sqrt{7}x + 1 = 0$ है, तो $x^5 + \frac{1}{x^5}$ का मान क्या होगा?

$$x + \frac{1}{x} = \sqrt{7}$$

$$x^2 + \frac{1}{x^2} = 7 - 2 = 5$$

$$x^3 + \frac{1}{x^3} = (\sqrt{7})^3 - 3\sqrt{7}$$

$$7\sqrt{7} - 3\sqrt{7}$$

$$= 4\sqrt{7}$$

$$\hookrightarrow \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$

$$5 \times 4\sqrt{7} - \sqrt{7}$$

$$20\sqrt{7} - \sqrt{7}$$

$$= 19\sqrt{7}$$

Q) If $x > 0$ and $x^4 + \frac{1}{x^4} = 254$, what is the value of $x^5 + \frac{1}{x^5}$?

यदि $x > 0$ है और $x^4 + \frac{1}{x^4} = 254$ है, तब $x^5 + \frac{1}{x^5}$ का मान कितना है ?

$$x^2 + \frac{1}{x^2} = \sqrt{254+2} = 16 \quad \downarrow \quad \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$

$$x + \frac{1}{x} = \sqrt{16+2} = \sqrt{18}$$

$$x^3 + \frac{1}{x^3} = (\sqrt{18})^3 - 3\sqrt{18} = 18\sqrt{18} - 3\sqrt{18} = 15\sqrt{18}$$

$$16 \times 15\sqrt{18} - \sqrt{18}$$

$$240\sqrt{18} - \sqrt{18}$$

$$239\sqrt{18}$$

$$9 \times 2$$

$$239 \times 3\sqrt{2}$$

$$717\sqrt{2}$$

$$\bullet \left(x^2 + \frac{1}{x^2}\right)$$

$$\boxed{x^6 + \frac{1}{x^6} = \left(x^2 + \frac{1}{x^2}\right)^3 - 3\left(x^2 + \frac{1}{x^2}\right)}$$

Ex:- $x + \frac{1}{x} = 3$ $x^6 + \frac{1}{x^6}$

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$$

$$\downarrow$$

$$\left(x^2 + \frac{1}{x^2}\right)^3 - 3\left(x^2 + \frac{1}{x^2}\right) = 7^3 - 3 \times 7 = 322$$

$$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3 = 18$$

$\downarrow$
Square

$$\left(x^3 + \frac{1}{x^3}\right)^2 = 18^2 - 2 = 322$$

$$\boxed{x^6 + \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right)^2 - 2}$$

Q) If $x + \frac{1}{x} = 7$, then the value of $x^6 + \frac{1}{x^6}$ is ?

यदि $x + \frac{1}{x} = 7$ है, तो $x^6 + \frac{1}{x^6}$ का मान क्या है ?

$$x^2 + \frac{1}{x^2} = 7^2 - 2$$

$$= 47$$

$$x^6 + \frac{1}{x^6} = 47^3 - 3 \times 47$$

$$47(47^2 - 3)$$

$$(2209 - 3) \times 47 = 2206 \times 47$$

$$103682$$

Q) If $(x + \frac{1}{x}) = 2\sqrt{2}$ and $x > 1$, what is the value of $(x^6 - \frac{1}{x^6})$?

यदि $(x + \frac{1}{x}) = 2\sqrt{2}$ है, और $x > 1$ तब $(x^6 - \frac{1}{x^6})$ का मान कितना है ?

$$\downarrow$$

$$x^3 + \frac{1}{x^3} = (2\sqrt{2})^3 - 3 \times 2\sqrt{2}$$

$$16\sqrt{2} - 6\sqrt{2}$$

$$= 10\sqrt{2}$$

$$x - \frac{1}{x} = \sqrt{x^2 - 4}$$

$$\sqrt{8-4}$$

$$= \sqrt{4} = 2$$

$$x^3 - \frac{1}{x^3} = 2^3 + 3 \times 2$$

$$= 14$$

$$\downarrow$$

$$(x^3 - \frac{1}{x^3})(x^3 + \frac{1}{x^3})$$

$$14 \times 10\sqrt{2}$$

$$140\sqrt{2}$$

Q) If $(y - \frac{1}{y}) = 4$, find the value of $(y^6 + \frac{1}{y^6})$

यदि $(y - \frac{1}{y}) = 4$, $(y^6 + \frac{1}{y^6})$ का मान ज्ञात कीजिए ।

$$\hookrightarrow y^2 + \frac{1}{y^2} = 4^2 + 2$$

$$= 18$$

$$y^6 + \frac{1}{y^6} = 18^3 - 3 \times 18$$

$$18 [18^2 - 3]$$

$$18 \times 321$$

$$5778$$

Q) If $\left(x - \frac{1}{x}\right) = 2\sqrt{2}$ and $x > 1$, what is the value of $\left(x^6 - \frac{1}{x^6}\right)$?

यदि $\left(x - \frac{1}{x}\right) = 2\sqrt{2}$ और $x > 1$ है, तो $\left(x^6 - \frac{1}{x^6}\right)$ का मान क्या है?

$$x^3 - \frac{1}{x^3} = (2\sqrt{2})^3 + 3 \times 2\sqrt{2}$$

$$16\sqrt{2} + 6\sqrt{2}$$

$$22\sqrt{2}$$

$$x + \frac{1}{x} = \sqrt{x^2 + \frac{1}{x^2}}$$

$$\sqrt{8+4}$$

$$= \sqrt{12}$$

$$x^3 + \frac{1}{x^3} = (\sqrt{12})^3 - 3 \times \sqrt{12}$$

$$12\sqrt{12} - 3\sqrt{12}$$

$$9\sqrt{12}$$

$$4 \times 3$$

$$18\sqrt{3}$$

$$\left(x^3 - \frac{1}{x^3}\right) \left(x^3 + \frac{1}{x^3}\right)$$

$$22\sqrt{2} \times 18\sqrt{3}$$

$$396\sqrt{6}$$

Q) If $x^2 + 3x + 1 = 0$, then the value of $x^6 + \frac{1}{x^6}$?

यदि $x^2 + 3x + 1 = 0$ हो, तो $x^6 + \frac{1}{x^6}$ का मान क्या है?

$$x + \frac{1}{x} + 3 = 0$$

$$\boxed{x + \frac{1}{x} = -3}$$

$$x^2 + \frac{1}{x^2} = (-3)^2 - 2$$

$$= 7$$

$$x^6 + \frac{1}{x^6} = 7^3 - 3 \times 7$$

$$= 322$$

$$\bullet \underline{x^3 \times x^4 = x^7}$$

$$\left(x^3 + \frac{1}{x^3}\right) \left(x^4 + \frac{1}{x^4}\right)$$

$$x^7 + \frac{1}{x} + x + \frac{1}{x^7}$$

$$x^7 + \frac{1}{x^7} + \left(x + \frac{1}{x}\right)$$

$$\boxed{x^7 + \frac{1}{x} = \left(x^3 + \frac{1}{x^3}\right)\left(x^4 + \frac{1}{x^4}\right) - \left(x + \frac{1}{x}\right)}$$

Q) If $x + \frac{1}{x} = 3$, $x \neq 0$, then the value of $x^7 + \frac{1}{x}$ is?

यदि $x + \frac{1}{x} = 3$, $x \neq 0$ है, तो $x^7 + \frac{1}{x}$ का मान क्या है?

$$\downarrow$$

$$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3$$

$$= 18$$

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$$

$$x^4 + \frac{1}{x^4} = 7^2 - 2 = 47$$

$$\left(x^3 + \frac{1}{x^3}\right)\left(x^4 + \frac{1}{x^4}\right) - \left(x + \frac{1}{x}\right)$$

$$18 \times 47 - 3$$

$$846 - 3$$

$$843$$

Q) If $x > 0$ and $x^4 + \frac{1}{x^4} = 142$, what is the value of $x^7 + \frac{1}{x}$?

यदि $x > 0$ और $x^4 + \frac{1}{x^4} = 142$ है, $x^7 + \frac{1}{x}$ का मान क्या है?

$$\downarrow$$

$$x^2 + \frac{1}{x^2} = \sqrt{142 + 2}$$

$$= 12$$

$$x + \frac{1}{x} = \sqrt{12 + 2} = \sqrt{14}$$

$$\downarrow$$

$$x^3 + \frac{1}{x^3} = (\sqrt{14})^3 - 3\sqrt{14}$$

$$14\sqrt{14} - 3\sqrt{14}$$

$$11\sqrt{14}$$

$$\left(x^3 + \frac{1}{x^3}\right)\left(x^4 + \frac{1}{x^4}\right) - \left(x + \frac{1}{x}\right)$$

$$11\sqrt{14} \times 142 - \sqrt{14}$$

$$1562\sqrt{14} - \sqrt{14}$$

$$1561\sqrt{14}$$

$$\bullet \left(x^4 + \frac{1}{x^4}\right)\left(x^3 + \frac{1}{x^3}\right)$$

$$\boxed{x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)}$$

$$x^8 + \frac{1}{x^8} \xrightarrow{\text{Given}} \left(x - \frac{1}{x}\right) \xrightarrow{\text{Step II}} \left(x^2 + \frac{1}{x^2}\right)$$

$$\xrightarrow{\text{Step III}} \left(x^4 + \frac{1}{x^4}\right) \xrightarrow{\text{Step IV}} x^8 + \frac{1}{x^8}$$

Q) If $x - \frac{1}{x} = 1$, then find the value of $x^8 + \frac{1}{x^8}$

यदि $x - \frac{1}{x} = 1$ है, तो $x^8 + \frac{1}{x^8}$ का मान ज्ञात करें।

$$\hookrightarrow x^2 + \frac{1}{x^2} = 1^2 + 2 = 3$$

$$x^4 + \frac{1}{x^4} = 3^2 - 2 = 7$$

$$x^8 + \frac{1}{x^8} = 7^2 - 2 = 47$$

Q) If $\left(x + \frac{1}{x}\right) = 5$ and $x > 1$, then the value of $\left(x^8 - \frac{1}{x^8}\right)$ is?

यदि $\left(x + \frac{1}{x}\right) = 5$ और $x > 1$ है, तो $\left(x^8 - \frac{1}{x^8}\right)$ का मान है।

$$x - \frac{1}{x} = \sqrt{5^2 - 4} = \sqrt{21}$$

$$x^2 + \frac{1}{x^2} = 5^2 - 2 = 23$$

$$x^4 + \frac{1}{x^4} = 23^2 - 2 = 527$$

$$\hookrightarrow \left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2}\right) \left(x^4 + \frac{1}{x^4}\right)$$

$$\sqrt{21} \times 5 \times 23 \times 527$$

$$12121 \times 5 \sqrt{21}$$

$$60605\sqrt{21}$$

1. If $\left(x + \frac{1}{x}\right) = 10$, what is the value of $\left(x^4 + \frac{1}{x^4}\right)$

यदि $\left(x + \frac{1}{x}\right) = 10$ है, तो $\left(x^4 + \frac{1}{x^4}\right)$ का मान क्या है?

(a) 9606

(b) 9600

(c) 9602

(d) 9604

2. If $x^2 - 4x + 4 = 0$, then the value of $16\left(x^4 - \frac{1}{x^4}\right)$ is.

यदि $x^2 - 4x + 4 = 0$, तब $16\left(x^4 - \frac{1}{x^4}\right)$ का मान है।

(a) 127

(b) $\frac{255}{16}$

(c) $\frac{127}{16}$

(d) 255

3. If $\left(x^2 + \frac{1}{x^2}\right) = 6$ and $0 < x < 1$, what is the value of $x^4 - \frac{1}{x^4}$?

यदि $\left(x^2 + \frac{1}{x^2}\right) = 6$ और $0 < x < 1$ है, $x^4 - \frac{1}{x^4}$ का मान क्या है?

(a) $-24\sqrt{2}$

(b) $-12\sqrt{10}$

(c) $24\sqrt{2}$

(d) $12\sqrt{10}$

4. If $\left(2x - \frac{3}{x}\right) = 2$, then find the value of $\left(16x^4 + \frac{81}{x^4}\right)$.

यदि $\left(2x - \frac{3}{x}\right) = 2$ है, तो $\left(16x^4 + \frac{81}{x^4}\right)$ का मान ज्ञात करें।

(a) 220

(b) 180

(c) 184

(d) 328

5. If $x^4 + \frac{16}{x^4} = 27217$ ($x > 0$), then find the value of $\left(x + \frac{2}{x}\right)$ is.

यदि $x^4 + \frac{16}{x^4} = 27217$ ($x > 0$) है, तो $\left(x + \frac{2}{x}\right)$ का मान ज्ञात करें।

(a) 13

(b) 15

(c) 11

(d) 17

6. If $x^4 + \frac{1}{x^4} = 194$, $x > 0$, then find the value of $x^3 + \frac{1}{x^3} + x + \frac{1}{x}$.

यदि $x^4 + \frac{1}{x^4} = 194$, $x > 0$, तो $x^3 + \frac{1}{x^3} + x + \frac{1}{x}$ का मान ज्ञात कीजिए।

(a) 56

(b) 66

(c) 76

(d) 46

7. If $\left(x + \frac{1}{x}\right) = 3\sqrt{2}$, then the value of $\left(x^5 + \frac{1}{x^5}\right)$ is.

यदि $\left(x + \frac{1}{x}\right) = 3\sqrt{2}$ है, तो $\left(x^5 + \frac{1}{x^5}\right)$ का मान है

- (a) $1581\sqrt{2}$
- (b) $789\sqrt{2}$
- (c) $689\sqrt{2}$
- (d) $717\sqrt{2}$

8. If $\left(y - \frac{1}{y}\right) = -9$, then what will be the value of $\left(y^5 - \frac{1}{y^5}\right)$?

यदि $\left(y - \frac{1}{y}\right) = -9$ हो, तो $\left(y^5 - \frac{1}{y^5}\right)$ का मान क्या होगा?

- (a) -62748
- (b) -62739
- (c) -62750
- (d) -59049

9. If $x^2 - 2\sqrt{5}x + 1 = 0$, then what is the value of $x^5 + \frac{1}{x^5}$?

यदि $x^2 - 2\sqrt{5}x + 1 = 0$ है, तो $x^5 + \frac{1}{x^5}$ का मान क्या है?

- (a) $408\sqrt{5}$
- (b) $612\sqrt{5}$
- (c) $406\sqrt{5}$
- (d) $610\sqrt{5}$

10. If $a + \frac{1}{a} = 3$, then what is $\left(a^6 + \frac{1}{a^6}\right)$ equal to?

यदि $a + \frac{1}{a} = 3$ है, तो $\left(a^6 + \frac{1}{a^6}\right)$ किसके समान होगा?

- (a) 730
- (b) 319
- (c) 322
- (d) 780

11. If $[x - (1/x)] = 2$, then what is the value of $[x^6 - (1/x^6)]$?

यदि $[x - (1/x)] = 2$ है, तो $[x^6 - (1/x^6)]$ का मान क्या है?

- (a) $140\sqrt{2}$
- (b) $134\sqrt{2}$
- (c) $114\sqrt{3} + 1$
- (d) $142\sqrt{2} + 3$

12. If $\left(x - \frac{1}{x}\right) = 2\sqrt{2}$, what is the value of $\left(x^6 + \frac{1}{x^6}\right)$?

यदि $\left(x - \frac{1}{x}\right) = 2\sqrt{2}$ है, तो $\left(x^6 + \frac{1}{x^6}\right)$ का मान कितना है?

- (a) 1000
- (b) 960
- (c) 1030
- (d) 970

13. If $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$, then find the value of $x^6 + \frac{1}{x^6}$.

यदि $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$ है, तो $x^6 + \frac{1}{x^6}$ का मान ज्ञात करें।

- (a) 2712
- (b) 2270
- (c) 2502
- (d) 2702

14. If $x + \frac{1}{x} = 4$, $x \neq 0$, then find the value of $x^7 + \frac{1}{x^7}$.

यदि $x + \frac{1}{x} = 4$, $x \neq 0$ है, तो $x^7 + \frac{1}{x^7}$ का मान बताइए।

- (a) 10084
(b) 11084
(c) 10196
(d) 11196

15. If $\left(x + \frac{1}{x}\right) = \sqrt{6}$ and $x > 1$, what is the value of $\left(x^8 - \frac{1}{x^8}\right)$?

यदि $\left(x + \frac{1}{x}\right) = \sqrt{6}$ और $x > 1$ है, $\left(x^8 - \frac{1}{x^8}\right)$ का मान क्या है?

- (a) $120\sqrt{3}$
(b) $108\sqrt{3}$
(c) $128\sqrt{3}$
(d) $112\sqrt{3}$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
C	D	A	C	A	A	D	B	D	C
11	12	13	14	15					
A	D	D	A	D					

Worksheet solution

Sol 1 By using formula

$$\text{If } x + \frac{1}{x} = a, \text{ then } x^4 + \frac{1}{x^4} = (a^2 - 2)^2 - 2$$

Ans.

$$x^4 + \frac{1}{x^4} = [(10)^2 - 2]^2 - 2$$

$$= 98^2 - 2$$

$$= 9604 - 2$$

$$= 9602 \underline{\text{Ans}}$$

Sol 2

Put $x=2$ (equation satisfied)

$$16 \left[x^4 - \frac{1}{x^4} \right] = 16 \left[16 - \frac{1}{16} \right]$$

$$= 256 - 1$$

$$= 255 \underline{\text{Ans}}$$

Sol 3

$$x^2 + \frac{1}{x^2} = 6$$

Now

$$x^4 - \frac{1}{x^4} = \left(x - \frac{1}{x} \right) \left(x + \frac{1}{x} \right)$$

$$\left(x^2 + \frac{1}{x^2} \right)$$

$$= \pm 2 \times 2\sqrt{2} \times 6$$

$$= -24\sqrt{2} \underline{\text{Ans}}$$

$$(\because 0 < x < 1)$$

Sol 4

$$\left(2x - \frac{3}{x} \right) = 2$$

Squaring both side

$$4x^2 + \frac{9}{x^2} = 4 + 12 = 16$$

Again squaring both side

$$16x^4 + \frac{81}{x^4} = 256 + 2 \times 4 \times 9$$

$$= 256 + 72$$

$$= 328 \underline{\text{Ans}}$$

Sol 5

$$x^4 + \frac{16}{x^4} = 27217$$

$$x^2 + \frac{4}{x^2} = \sqrt{27217 + 8}$$

$$= \sqrt{27225}$$

$$= 165$$

$$\therefore x + \frac{2}{x} = \sqrt{165 + 4}$$

$$= \sqrt{169} = 13 \underline{\text{Ans}}$$

Sol 6

$$x + \frac{1}{x^4} = (a^2 - 2)^2 - 2$$

$$194 = (a^2 - 2)^2 - 2$$

$$(a^2 - 2)^2 = 196$$

$$a = 4$$

$\therefore$

$$x^3 + \frac{1}{x^3} + x + \frac{1}{x}$$

$$= 4^3 - 3 \times 4 + 4$$

$$= 64 - 12 + 4$$

$$= 56 \underline{\text{Ans}}$$

Sol 7 Formula-

$$x + \frac{1}{x} = a \text{ then,}$$

$$x^5 + \frac{1}{x^5} = (a^2 - 2)(a^3 - 3a) - a$$

ATQ

$$x^5 + \frac{1}{x^5} = [(3\sqrt{2})^2 - 2]$$

$$[(3\sqrt{2})^3 - 3 \times 3\sqrt{2}] - 3\sqrt{2}$$

$$= 16 \times 45\sqrt{2} - 3\sqrt{2}$$

$$= 717\sqrt{2} \underline{\underline{Ans}}$$

Sol 8

ATQ

$$y^5 - \frac{1}{y^5} = [(-9)^2 + 1]$$

$$[(-9)^3 + 4 \times (-9)] + (-9)$$

$$= 82 \times (-765) - 9$$

$$= -62739 \underline{\underline{Ans}}$$

Sol 9

$$x^2 - 2\sqrt{5}x + 1 = 0$$

संप्रीकरण को x से भाग देने पर-

$$x + \frac{1}{x} = 2\sqrt{5}$$

$$x^5 + \frac{1}{x^5} = [(2\sqrt{5})^2 - 2]$$

$$[(2\sqrt{5})^3 - 3(2\sqrt{5})] - 2\sqrt{5}$$

$$= 18 \times 32\sqrt{5} - 2\sqrt{5}$$

$$= 612\sqrt{5} - 2\sqrt{5}$$

$$= 610\sqrt{5} \underline{\underline{Ans}}$$

Sol 10 By using Formula

$$\text{If } x + \frac{1}{x} = a \text{ then}$$

$$x^6 + \frac{1}{x^6} = (a^3 - 3a)^2 - 2$$

$$\therefore x^6 + \frac{1}{x^6} = (3^3 - 3 \times 3)^2 - 2$$

$$= (27 - 9)^2 - 2$$

$$= 18^2 - 2$$

$$= 324 - 2$$

$$= 322 \underline{\underline{Ans}}$$

Sol 11

ATQ

$$x - \frac{1}{x} = 2$$

$$x + \frac{1}{x} = \sqrt{(2)^2 + 4}$$

$$= 2\sqrt{2}$$

Now,

$$(x^6 - \frac{1}{x^6}) = (x^3 + \frac{1}{x^3})$$

$$(x^3 - \frac{1}{x^3})$$

$$= [(2\sqrt{2})^3 - 3 \times 2\sqrt{2}]$$

$$[(2)^3 + 3 \times 2]$$

$$= 10\sqrt{2} \times 14$$

$$= 140\sqrt{2} \underline{\underline{Ans}}$$

Sol 12 ATQ

$$x - \frac{1}{x} = 2\sqrt{2}$$

$$x + \frac{1}{x} = \sqrt{(2\sqrt{2})^2 + 4}$$

$$= 2\sqrt{3}$$

$$\therefore x^6 + \frac{1}{x^6} = \left[(2\sqrt{3})^3 - 3 \times 2\sqrt{2} \right]^2 - 2$$

$$= (18\sqrt{3})^2 - 2$$

$$= 972 - 2$$

$$= 970 \text{ Ans}$$

Sol 13

ATQ

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 = (\sqrt{6})^2$$

$$x + \frac{1}{x} = 6 - 2 = 4$$

$$\therefore x^6 + \frac{1}{x^6} = [(4)^3 - 3 \times 4]^2 - 2$$

$$= [64 - 12]^2 - 2$$

$$= 52^2 - 2$$

$$= 2704 - 2$$

$$= 2702 \text{ Ans}$$

Sol 14 ATQ

$$x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4$$

$$= 64 - 12 = 52$$

$$x^2 + \frac{1}{x^2} = 4^2 - 2$$

$$= 14$$

$$x^4 + \frac{1}{x^4} = 14^2 - 2$$

$$= 194$$

$$\therefore x^7 + \frac{1}{x^7} = \left(x^3 + \frac{1}{x^3} \right)$$

$$\times \left(x^4 + \frac{1}{x^4} \right) -$$

$$\left(x + \frac{1}{x} \right)$$

$$= 52 \times 194 - 4$$

$$= 10084 \text{ Ans}$$

Sol 15

$$x + \frac{1}{x} = \sqrt{6}$$

$$x - \frac{1}{x} = \sqrt{(\sqrt{6})^2 - 4} = \sqrt{2}$$

$$x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4} \right) \left(x^2 + \frac{1}{x^2} \right)$$

$$\left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)$$

$$= \left[\{ (\sqrt{6})^2 - 2 \}^2 - 2 \right] \times 4$$

$$\times \sqrt{6} \times \sqrt{2}$$

$$= 14 \times 8\sqrt{2}$$

$$= 112\sqrt{3} \text{ Ans}$$