

QUADRATIC EQUATION

Q) If α and β are the roots of the quadratic equation $2x^2 + 6x + k = 0$, where $k < 0$, then what is the maximum value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$?

यदि α और β द्विघात समीकरण $2x^2 + 6x + k = 0$ के मूल हैं, जहाँ $k < 0$ है, तो $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ का अधिकतम मान क्या है?

$$2x^2 + 6x + k = 0$$

$$\alpha \cdot \beta = \frac{k}{2}$$

$$\alpha + \beta = \frac{-6}{2} = -3$$

$$\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$\frac{(\alpha + \beta)^2}{\alpha\beta} - 2$$

$$\frac{(-3)^2 \times 2}{k} - 2$$

$$\left[\frac{18}{k} - 2 \right]$$

$$\frac{18}{k} < 0$$

$$\frac{18}{k} - 2 < \textcircled{-2} \text{ Max value}$$

Q) If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$ is?

यदि α और β समीकरण $ax^2 + bx + c = 0$ के मूल हैं, तो $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$ का मान क्या है?

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\frac{a\beta + b + a\alpha + b}{(a\alpha + b)(a\beta + b)}$$

$$\frac{a(\alpha + \beta) + 2b}{a^2\alpha\beta + ab\alpha + ab\beta + b^2}$$

$$\frac{a(\alpha + \beta) + 2b}{a^2\alpha\beta + ab(\alpha + \beta) + b^2}$$

$$\frac{\alpha \times \left(-\frac{b}{\alpha}\right) + 2b}{a^2\left(\frac{c}{\alpha}\right) + ab\left(-\frac{b}{\alpha}\right) + b^2}$$

$$\frac{b}{ac - b^2 + b^2} = \frac{b}{ac}$$

Q) If α and β are the roots of the quadratic equation $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + (8+2\sqrt{5}) = 0$, then the value of $2\alpha\beta/(\alpha+\beta)$ is .

यदि α तथा β द्विघात समीकरण $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + (8+2\sqrt{5})$ के मूल हों, तो $2\alpha\beta/(\alpha+\beta)$ का मान ज्ञात कीजिए।

$$\alpha + \beta = \frac{4+\sqrt{5}}{5+\sqrt{2}}$$

$$\alpha \cdot \beta = \frac{8+2\sqrt{5}}{5+\sqrt{2}}$$

$$\frac{2 \times 2(4+\sqrt{5})}{(5+\sqrt{2})}$$

$$\frac{4+\sqrt{5}}{5+\sqrt{2}}$$

$$2 \times 2 = 4$$

TYPE-IV

One Root $\rightarrow a + \sqrt{b}$
Other Root $\rightarrow a - \sqrt{b}$

* One Root $\rightarrow 2+\sqrt{3}$
Quadratic eqⁿ = ?

$$\alpha = 2+\sqrt{3}$$

$$\beta = 2-\sqrt{3}$$

$$\alpha + \beta = 4$$

$$\alpha \cdot \beta = (2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$$

$$x^2 - (\alpha + \beta)x + \alpha \cdot \beta = 0$$

$$x^2 - 4x + 1 = 0$$

Q) Quadratic equation corresponding to the roots $2+\sqrt{5}$ and $2-\sqrt{5}$ is .
 $2+\sqrt{5}$ और $2-\sqrt{5}$ मूलों के अनुरूप द्विघात समीकरण है।

$$\alpha + \beta = 4$$

$$\alpha \cdot \beta = (2+\sqrt{5})(2-\sqrt{5})$$

$$4-5 = -1$$

$$x^2 - (\alpha + \beta)x + \alpha \beta = 0$$

$$x^2 - 4x + (-1)$$

$$x^2 - 4x - 1 = 0$$

Q) One root of the equation $5x^2 + 2x + Q = 2$ is reciprocal of another.
What is the value of Q^2 ?

समीकरण $5x^2 + 2x + Q = 2$ का एक मूल दूसरे का व्युत्क्रम है। Q^2 का मान क्या है?

$$5x^2 + 2x + Q - 2 = 0$$

$$\alpha, \beta$$

$$\beta = \frac{1}{\alpha}$$

$$\alpha \cdot \beta = \frac{Q-2}{5}$$

$$\alpha \cdot \frac{1}{\alpha} = \frac{Q-2}{5}$$

$$Q = 7$$

$$Q^2 = 7^2 = 49$$

Q) If α and β are the roots of equation $x^2 - x + 1 = 0$ then which equation will have root α^3 and β^3 ?

यदि α तथा β समीकरण $x^2 - x + 1 = 0$ के मूल हैं, तो किस समीकरण के मूल α^3 तथा β^3 होंगे?

$$(x+1)(x^2-x+1) = 0 \times (x+1)$$

$$x^3 + 1^3 = 0$$

$$\boxed{x^3 = -1}$$

$$\downarrow$$

$$\alpha^3 = \beta^3 = -1$$

$$x^2 - (\alpha^3 + \beta^3)x + \alpha^3 \cdot \beta^3$$

$$x^2 - (-2)x + 1$$

$$x^2 + 2x + 1 = 0$$

$$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$

$$\alpha^3 + \beta^3 = -1 - 1 = -2$$

$$\alpha^3 \cdot \beta^3 = (-1)(-1) = 1$$

Q) If α and β are the roots of equation $x^2 - x + 1 = 0$ then which equation will have root α^{30} and β^{30} ?

यदि α तथा β समीकरण $x^2 - x + 1 = 0$ के मूल हैं, तो किस समीकरण के मूल α^{30} तथा β^{30} होंगे?

$$(x+1)(x^2-x+1) = 0 \times (x+1)$$

$$x^3 + 1^3 = 0$$

$$\boxed{x^3 = -1}$$

$$\downarrow$$

$$x^3 = \beta^3 = -1$$

$$x^2 - (1+1)x + 1 \times 1$$

$$x^2 - 2x + 1 = 0$$

$$\alpha^{30} = (\alpha^3)^{10} = (-1)^{10} = 1$$

$$\beta^{30} = (\beta^3)^{10} = (-1)^{10} = 1$$

Q) If α and β are the roots of the equation $x^2 + x - 1 = 0$, then what is the equation whose roots are α^5 and β^5 ?

यदि α तथा β समीकरण $x^2 + x - 1 = 0$ के मूल हैं, तो वह समीकरण क्या है, जिसके मूल α^5 तथा β^5 होंगे?

$$\alpha + \beta = -1$$

$$\alpha \cdot \beta = -1$$

$$\boxed{\beta = -\frac{1}{\alpha}}$$

$$\boxed{\alpha - \frac{1}{\alpha} = -1}$$

$$\alpha^5 \text{ and } \beta^5$$

$$\alpha^5 + \left[-\frac{1}{\alpha^5}\right]$$

$$\text{Sum of root} \left[\alpha^5 - \frac{1}{\alpha^5} \right]$$

$$\begin{aligned} & [(-1)^3 - 3(-1)] [(-1)^2 + 2] - (-1) \\ & [-1 + 3] 3 + 1 \\ & 2 \times 3 + 1 \\ & = 7 \end{aligned}$$

Product of root

$$\alpha^5 \cdot \frac{1}{\alpha^5}$$

$$= -1$$

$$x^2 - 7x - 1 = 0$$

$$a^5 - \frac{1}{a^5} = \left(a^3 - \frac{1}{a^3}\right) \left(a^2 + \frac{1}{a^2}\right) - \left(a - \frac{1}{a}\right)$$

$$a - \frac{1}{a} = k$$

$$a^2 + \frac{1}{a^2} = k^2 + 2$$

$$a - \frac{1}{a} = k$$

$$a^3 - \frac{1}{a^3} = k^3 + 3k$$

Q) If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then what is the equation whose roots are $\frac{\alpha^3}{\beta^2}$ and $\frac{\beta^3}{\alpha^2}$?

यदि α तथा β समीकरण $x^2 - 2x + 4 = 0$ के मूल हैं, तो वह समीकरण क्या है जिसके मूल $\frac{\alpha^3}{\beta^2}$ तथा $\frac{\beta^3}{\alpha^2}$?

$$x^2 - 2x + 4 = 0$$

$$(x+2)(x^2 - 2xx + 2^2) = 0 \times (x+2)$$

$$x^3 + 2^3 = 0$$

$$x^3 = -8$$

$$\alpha^3 = \beta^3 = -8$$

$$\frac{\alpha^3}{\beta^2} \text{ and } \frac{\beta^3}{\alpha^2}$$

$$\alpha + \beta = 2$$

$$\alpha \cdot \beta = 4$$

$$\begin{array}{cc} \frac{\cancel{\beta} \alpha^3}{\cancel{\beta}^3} & \frac{\cancel{\alpha} \beta^3}{\cancel{\alpha}^2} \\ \downarrow & \downarrow \\ \beta & \alpha \end{array}$$

$$\text{Sum of roots} = \beta + \alpha = 2$$

$$\text{Product} = \beta \cdot \alpha = 4$$

$$x^2 - (\alpha + \beta)x + \alpha \cdot \beta$$

$$x^2 - 2x + 4 = 0$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$$

$$(a+b)^2 - (a-b)^2 = 4ab$$

$$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

Q) α and β are the roots of quadratic equation. If $\alpha + \beta = 8$ and $\alpha - \beta = 2\sqrt{5}$, then which of the following equation will have roots α^4 and β^4 ?
 यदि α तथा β द्विघात समीकरण के मूल हैं। यदि $\alpha + \beta = 8$ तथा $\alpha - \beta = 2\sqrt{5}$ हैं, तो α^4 तथा β^4 निम्नलिखित में से किस समीकरण के मूल हैं।

$$\alpha + \beta = 8$$

$$\alpha - \beta = 2\sqrt{5}$$

$$(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$$

$$8^2 - (2\sqrt{5})^2 = 4\alpha\beta$$

$$64 - 20$$

$$\Rightarrow \frac{44}{4} = \alpha\beta$$

$$\boxed{\alpha \cdot \beta = 11}$$

$$(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\begin{array}{l} 8^2 - 2 \times 11 \\ 64 - 22 \\ \hline 42 \end{array}$$

$$\alpha^4 \text{ तथा } \beta^4$$

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$$

$$(42)^2 - 2 \times 11^2$$

$$1764 - 242$$

$$\text{Sum of Roots} = 1522$$

$$\text{Product} = \alpha^4 \cdot \beta^4 = 11^4$$

$$14641$$

$$x^2 - 1522x + 14641 = 0$$

Q) If the difference between the roots of the equation $Ax^2 - Bx + C = 0$ is 4, then which of the following is True?

यदि समीकरण $Ax^2 - \beta x + C = 0$ के मूलों का अंतर 4 है, तो निम्नलिखित में से कौन-सा सत्य है?

$$\alpha - \beta = 4$$

$$\alpha \cdot \beta = \frac{C}{A}$$

$$\alpha + \beta = \frac{-(-\beta)}{A} = \frac{\beta}{A}$$

$$(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$$

$$\left(\frac{\beta}{A}\right)^2 - 4^2 = 4 \times \frac{C}{A}$$

$$A^2 \times \left(\frac{\beta^2}{A^2} - 16\right) = \frac{4C}{A} \times A^2$$

$$\beta^2 - 16A^2 = 4AC$$

$$\beta^2 - 10A^2 - 6A^2 = 4AC$$

$$\beta^2 - 10A^2 = 4AC + 6A^2$$

Q) α and β are the roots of the quadratic equation $x^2 - x - 1 = 0$. What is the value of $\alpha^8 + \beta^8$?

यदि α तथा β द्विघात समीकरण $x^2 - x - 1 = 0$ के मूल हैं। $\alpha^8 + \beta^8$ का मान क्या है?

$$\alpha + \beta = 1$$

$$\alpha \cdot \beta = -1$$

$$\beta = \frac{-1}{\alpha}$$

$$\alpha - \frac{1}{\alpha} = 1$$

$$\alpha^8 + \beta^8$$

$$\alpha^8 + \left[\frac{-1}{\alpha}\right]^8$$

$$\alpha^8 + \frac{1}{\alpha^8}$$

$$\alpha - \frac{1}{\alpha} = 1$$

$$\alpha^2 + \frac{1}{\alpha^2} = 1^2 + 2 = 3$$

$$\alpha^4 + \frac{1}{\alpha^4} = 3^2 - 2 = 7$$

$$\left\{ \begin{array}{l} \alpha - \frac{1}{\alpha} = k \\ \alpha^2 + \frac{1}{\alpha^2} = k^2 + 2 \\ \alpha^4 + \frac{1}{\alpha^4} = (k^2 + 2)^2 - 2 \\ \alpha^8 + \frac{1}{\alpha^8} \end{array} \right\}$$

$$\alpha^8 + \frac{1}{\alpha^8} = 7^2 - 2$$

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Q) If α and β are the roots of the equation $x^2 - x + 3 = 0$, then find the value of $\alpha^4 + \beta^4$?

यदि α तथा β समीकरण $x^2 - x + 3 = 0$ के मूल हैं, तो $\alpha^4 + \beta^4$ का मान क्या होगा?

$$\alpha + \beta = 1$$

$$\alpha \cdot \beta = 3$$

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\alpha^2 + \beta^2 = 1^2 - 2 \times 3$$

$$= -5$$

$$\alpha^4 + \beta^4 = (-5)^2 - 2 \times 3^2$$

$$25 - 18$$

7

Q) If α and β are the roots of the quadratic equation $x^2 - 6x + 2 = 0$ then what will be the value of $\alpha^4\beta^3 + \alpha^3\beta^4$ is?

यदि α एवं β द्विघातीय समीकरण $x^2 - 6x + 2 = 0$ के मूल हों, तो $\alpha^4\beta^3 + \alpha^3\beta^4$ का मान क्या होगा?

$$\alpha + \beta = 6$$

$$\alpha \cdot \beta = 2$$

$$\alpha^4\beta^3 + \alpha^3\beta^4$$

$$\alpha^3\beta^3(\alpha + \beta)$$

$$2^3(6)$$

$$8 \times 6 = 48$$

Q) If α and β are the roots of the quadratic equation $x^2 - 5x + 4 = 0$, then what will be the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta} + 2\alpha\beta\right)$ is

यदि α एवं β द्विघातीय समीकरण $x^2 - 5x + 4 = 0$ के मूल हों, तो $\left(\frac{1}{\alpha} + \frac{1}{\beta} + 2\alpha\beta\right)$ का मान होगा।

$$\alpha + \beta = 5$$

$$\alpha \cdot \beta = 4$$

$$\frac{\alpha + \beta}{\alpha \beta} = 2\alpha\beta$$

$$\frac{5}{4} = 2 \times 4$$

$$\frac{5}{4} = 8 \Rightarrow \frac{27}{4}$$

Q) If one of the roots of the quadratic equation: $4x^2 - 2x + (k-4) = 0$ is the reciprocal of the other root, then k is equal to:

यदि द्विघात समीकरण: $4x^2 - 2x + (k-4) = 0$ का एक मूल दूसरे मूल का व्युत्क्रम है, तो k बराबर है।

$\alpha \quad \beta$

$$\boxed{\beta = \frac{1}{\alpha}}$$

$$\alpha \cdot \beta = \frac{k-4}{4}$$

$$\cancel{\alpha} \cdot \frac{1}{\cancel{\alpha}} = \frac{k-4}{4}$$

$$k = 8$$

Q) If m and n are the roots of the equation $ax^2 + bx + c = 0$, then find the equation whose roots are $(m^2+1)/m$ and $(n^2+1)/n$ is.

यदि समीकरण $ax^2 + bx + c = 0$ के मूल m और n हैं, तो वह समीकरण ज्ञात कीजिए, जिसके मूल $(m^2+1)/m$ और $(n^2+1)/n$ हैं।

$$ax^2 + bx + c = 0$$

$$m + n = -\frac{b}{a}$$

$$m \cdot n = \frac{c}{a}$$

$$\left. \begin{array}{l} m + n = -\frac{b}{a} \\ m \cdot n = \frac{c}{a} \end{array} \right\} \begin{array}{l} m^2 + n^2 \\ = (m+n)^2 - 2mn \end{array}$$

Sum of roots

$$\frac{m^2+1}{m} + \frac{n^2+1}{n}$$

$$\frac{nm^2+n+mn^2+m}{mn}$$

$$\frac{mn(m+n) + (m+n)}{mn}$$

$$\frac{(m+n)(mn+1)}{mn}$$

$$\begin{aligned} \frac{-\frac{b}{a} \left[\frac{c}{a} + 1 \right]}{\frac{c}{a}} &\Rightarrow -\frac{b}{c} \left[\frac{c+a}{a} \right] \\ &\Rightarrow -\frac{b}{ac} (a+c) \\ &= -\frac{[ab+bc]}{ac} \end{aligned}$$

Product of Roots

$$\begin{aligned} \frac{m^2+1}{m} \times \frac{n^2+1}{n} \\ \frac{m^2n^2+m^2+n^2+1}{mn} \end{aligned}$$

$$m^2+n^2 = \left(-\frac{b}{a}\right)^2 - 2 \times \frac{c}{a}$$

$$a^2 \left[\frac{c^2}{a^2} + \frac{b^2}{a^2} - \frac{2c}{a} + 1 \right]$$

$$a^2 \times \frac{c}{a}$$

$$\frac{c^2+b^2-2ac+a^2}{ac}$$

$$\left[\frac{(c-a)^2+b^2}{ac} \right]$$

$$acx^2 + (ab+bc)x + b^2 + (a-c)^2 = 0$$

MISCELLANEOUS

Q) If $3x^2 - ax + 6 = ax^2 + 2x + 2$ has only one (repeated) solution, then the positive integral solution of a is:

यदि $3x^2 - ax + 6 = ax^2 + 2x + 2$ का केवल एक (दोहराया गया) हल है, तो a का धनात्मक समाकलन हल है;

$$3x^2 - ax^2 - ax - 2x + 6 - 2 = 0$$

$$(3-a)x^2 - (a+2)x + 4 = 0$$

$$D = 0 \text{ (for one repeated soln)}$$

$$b^2 - 4ac = 0$$

$$(a+2)^2 = 4 \times (3-a) \times 4$$

$$a^2 + 4 + 4a = 48 - 16a$$

$$a^2 + 20a - 44 = 0$$

$$\begin{array}{cc} -22 & +2 \\ \times & \checkmark \end{array}$$

Q) What is the value of α ($\alpha \neq 0$) for which $x^2 - 5x + \alpha$ and $x^2 - 7x + 2\alpha$ have a common factor?

α ($\alpha \neq 0$) का वह मान क्या है, जिसके लिए $x^2 - 5x + \alpha$ और $x^2 - 7x + 2\alpha$ का एक सार्व गुणनखंड होता है?

$$x^2 - 5x + \alpha$$

↓

$$\alpha, \beta$$

$$\alpha + \beta = 5 \quad \text{--- (3)}$$

$$\alpha \cdot \beta = \alpha \quad \text{--- (4)}$$

$$x^2 - 7x + 2\alpha$$

↓

$$\alpha, \delta$$

$$\alpha + \delta = 7 \quad \text{--- (1)}$$

$$\alpha \cdot \delta = 2\alpha \quad \text{--- (2)}$$

$$\frac{(2)}{(4)} = \frac{\alpha \cdot \delta}{\alpha \cdot \beta} = \frac{2\alpha}{\alpha}$$

$$\boxed{\delta = 2\beta}$$

$$\alpha + 2\beta = 7$$

$$\alpha + \beta = 5$$

$$\underline{\beta = 7 - 5 = 2}$$

$$\boxed{\alpha = 5 - 2 = 3}$$

$$\alpha \cdot \beta = \alpha$$

$$3 \times 2 = \alpha = 6$$

Q) If α and β are roots of the equation $x^2 + bx + c = 0$, then which equation will have roots $(\alpha\beta + \alpha + \beta)$ and $(\alpha\beta - \alpha - \beta)$?

यदि α तथा β समीकरण $ax^2 + bx + c = 0$ के मूल हैं, तो किस समीकरण के मूल $(\alpha\beta + \alpha + \beta)$ तथा $(\alpha\beta - \alpha - \beta)$ होंगे?

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\text{योग} = \alpha \cdot \beta + \alpha + \beta + \alpha \cdot \beta - \alpha - \beta$$

$$= 2\alpha\beta$$

$$2 \times \frac{c}{a}$$

$$a^2x^2 - 2acx + c^2 - b^2 = 0$$

$$\hookrightarrow \text{Sum} = -\left[\frac{2ac}{a^2}\right]$$

$$= -\frac{2c}{a}$$

Q) If the sum of the roots of a quadratic equation is -7 and product of roots is 12, then find the quadratic equation.

यदि एक द्विघात समीकरण के मूलों का योग -7 है और मूलों का गुणनफल 12 है, तो उस द्विघात समीकरण को ज्ञात करें।

$$x^2 - (-7)x + 12$$

$$x^2 + 7x + 12 = 0$$

Q) What will be the value of k so that the sum and product of the roots of the equation $3x^2 + (2k+1)x - k-5 = 0$ are equal?

k का मान क्या होगा जिससे की समीकरण $3x^2 + (2k+1)x - k-5 = 0$ के मूलों के योगफल एवं गुणनफल समान हो?

$$\left. \begin{aligned} \alpha + \beta &= -\frac{(2k+1)}{3} \\ \alpha \cdot \beta &= -\frac{(k+5)}{3} \end{aligned} \right\} \frac{-(2k+1)}{3} = \frac{-(k+5)}{3}$$

$$2k - k = 5 - 1$$

$$k = 4$$

$$3x^2 + (2k+1)x - (k+5) = 0$$

Q) If α, β are the roots of the quadratic equation $x^2 - q(1+x) - r = 0$, then the value of $(1+\alpha)(1+\beta)$ is?

यदि α, β द्विघात समीकरण $x^2 - q(1+x) - r = 0$ के मूल हों, तो $(1+\alpha)(1+\beta)$ का मान क्या होगा?

$$1 + \beta + \alpha + \alpha\beta$$

$$(1 + \alpha + \beta + \alpha\beta)$$

$$1 + q - q - r$$

$$= 1 - r$$

$$x^2 - q - qx - r = 0$$

$$x^2 - qx - (r+q) = 0$$

$$\alpha + \beta = q$$

$$\alpha \cdot \beta = -(q+r)$$

Q) If a root of the equation $x^2 + px + 12$ is 4, while the roots of the equation $x^2 + px + q = 0$ are equal, then the value of q will be:

यदि समीकरण $x^2 + px + 12 = 0$ का एक मूल 4 हो, जबकि समीकरण $x^2 + px + q = 0$ के मूल समान हों, तो q का मान होगा?

↳ $D = 0$

$\beta = 4$

$$x^2 + px + 12 = 0$$

$$\alpha + \beta = -p$$

$$\boxed{\alpha = -p - 4}$$

$$b^2 - 4ac = 0$$

$$b^2 = 4ac$$

$$p^2 = 4 \times 1 \times q$$

$$p^2 = 4q$$

$$\alpha \cdot \beta = 12$$

$$\cancel{\alpha}x - (p + 4) = 12$$

$$p + 4 = -3$$

$$p = -3 - 4 = -7$$

$$(-7)^2 = 4q$$

$$q = \frac{49}{4}$$

Q) If α and β are the roots of the equation $(x-a)(x-b) = c$, $c \neq 0$, then the roots of the equation $(x-\alpha)(x-\beta) + c = 0$ are

यदि α और β समीकरण $(x-a)(x-b) = c$, $c \neq 0$ के मूल हैं; तो समीकरण $(x-\alpha)(x-\beta) + c = 0$ के मूल हैं।

$$(x-a)(x-b) - c = 0$$

$$(x-a)(x-b) - c = (x-\alpha)(x-\beta)$$

$$(x-\alpha)(x-\beta) + c = (x-a)(x-b)$$

$$x = a \quad x = b$$

1. If the equation $ax^2 - 5x + c = 0$ has 10 as the sum of the roots and also as the product of the roots, which of the following is true?
यदि समीकरण $ax^2 - 5x + c = 0$ में मूलों का योग 10 है और मूलों का गुणनफल भी 10 है, तो निम्नलिखित में से कौन सा सत्य है?

(a) $a = \frac{1}{2}, c = 5$

(b) $a = 2, c = 3$

(c) $a = 5, c = \frac{1}{2}$

(d) $a = 3, c = 2$

2. If α, β are the roots of $x^2 + px + q = 0$, the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is:

यदि α, β समीकरण $x^2 + px + q = 0$ के मूल हैं, तो $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ का मान है

(c) $\frac{p^2 - 2q}{q}$

(b) $\frac{p^2 + 2q}{q}$

(c) $\frac{-p^2 + 2q}{q}$

(d) $\frac{-p^2 - 2q}{q}$

3. If α and β are the roots of the quadratic equation $x^2 + \alpha x + \beta = 0$, where $\beta \neq 0$, then what is the value of $\alpha - \beta$?

यदि α और β द्विघात समीकरण $x^2 + \alpha x + \beta = 0$ के मूल हैं, जहाँ $\beta \neq 0$, तो $\alpha - \beta$ का मान क्या है?

(a) 4

(b) 3

(c) -1

(d) -3

4. If the roots of the equation $px^2 + x + r = 0$ are reciprocal to each other, then which one the following is correct

यदि समीकरण $px^2 + x + r = 0$ के मूल एक-दूसरे के व्युत्क्रम में हैं, तो निम्नलिखित में से कौन-सा सही है?

(a) $p = 2r$

(b) $p = r$

(c) $2p = r$

(d) $p = 4r$

5. Find the quadratic equation whose roots are -5 & -3

वह समीकरण ज्ञात करो जिसके मूल $-5, -3$ हों।

(a) $x^2 + 4x + 15 = 0$

(b) $x^2 + 8x + 15 = 0$

(c) $x^2 + 6x + 15 = 0$

(d) None of these

6. Find the quadratic equation with rational coefficient having one roots is $5 + \sqrt{2}$

परिमेय गुणांक वाली द्विघात समीकरण ज्ञात कीजिए जिसका एक मूल $5 + \sqrt{2}$ हो।

(a) $2x^2 - 10x + 23 = 0$

(b) $x^2 - 10x + 23 = 0$

(c) $x^2 - 12x + 21 = 0$

(d) $5x^2 - 10x + 23 = 0$

7. If the roots of the equation $5x^2 - 7x + K = 0$ are reciprocal of each other. Find K .

यदि समीकरण $5x^2 - 7x + K = 0$ के मूल एक-दूसरे के व्युत्क्रम हैं तो K का मान ज्ञात कीजिए।

(a) 3

(b) 5

(c) 7

(d) 9

8. If α, β are the roots of the equation $2x^2 - 5x + 7 = 0$, then the equation whose roots are $(2\alpha + 3\beta)$ and $(3\alpha + 2\beta)$ is:

यदि α, β समीकरण $2x^2 - 5x + 7 = 0$ के मूल हैं, तो वह समीकरण जिसकी मूल $(2\alpha + 3\beta)$ और $(3\alpha + 2\beta)$ हैं:

(a) $2x^2 + 25x + 82 = 0$

(b) $2x^2 - 25x - 82 = 0$

(c) $2x^2 - 25x + 82 = 0$

(d) $2x^2 + 25x - 82 = 0$

9. The quadratic equation whose roots are the reciprocals of the roots of the equation $3x^2 - 20x + 17 = 0$, is:

वह द्विघात समीकरण जिसके मूल समीकरण $3x^2 - 20x + 17 = 0$ के मूलों के व्युत्क्रम हैं:

(a) $20x^2 - 17x + 3 = 0$

(b) $17x^2 - 20x + 3 = 0$

(c) $20x^2 + 17x - 3 = 0$

(d) $17x^2 + 20x - 3 = 0$

10. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$, find the equation whose roots are $\frac{\alpha^3}{\beta^2}, \frac{\beta^3}{\alpha^2}$.

यदि α, β समीकरण $x^2 - 2x + 4 = 0$ के मूल हैं। तो वह समीकरण ज्ञात करो जिसके मूल $\frac{\alpha^3}{\beta^2}, \frac{\beta^3}{\alpha^2}$ हों।

(a) $x^2 - 4x + 8 = 0$

(b) $x^2 - 2x + 4 = 0$

(c) $x^2 - 32x + 4 = 0$

(d) $x^2 - 16x + 4 = 0$

11. If α, β are the roots of the equation $x^2 + x + 1 = 0$, find the equation whose roots are α^{31}, β^{19}

यदि α, β समीकरण $x^2 + x + 1 = 0$ के मूल हैं। तो वह समीकरण ज्ञात करो जिसके मूल α^{31}, β^{19} हों।

- (a) $x^2 - x + 1 = 0$ (b) $x^2 + x + 1 = 0$
(c) $2x^2 - 3x + 1 = 0$ (d) $6x^2 - 9x + 2 = 0$

12. Two solutions of a quadratic equation are given as $x = \frac{4}{3}$ and $x = -\frac{3}{7}$. In what form equation can be written?

एक द्विघात समीकरण के दो हल $x = \frac{4}{3}$ और $x = -\frac{3}{7}$ के रूप में दिए गए हैं। समीकरण को किस रूप में लिखा जा सकता है?

- (a) $(7x + 3)(3x - 4) = 0$ (b) $(7x - 3)(3x + 4) = 0$
(c) $(7x + 3)(3x + 4) = 0$ (d) $(7x - 3)(3x + 4) = 0$

13. Find the positive value of m which the roots of the equation $12x^2 + mx + 5 = 0$ are in the ratio 3:2.

m का धनात्मक मान ज्ञात कीजिए यदि समीकरण $12x^2 + mx + 5 = 0$ के मूलों में 3:2 का अनुपात हो।

- (a) $m = 5\sqrt{5}$ (b) $m = 5\sqrt{10}$
(c) $m = \sqrt{10}$ (d) $m = \sqrt{5}$

14. The roots of the equation $4^x - 3 \cdot 2^{x+2} + 32 = 0$ would include. $4^x - 3 \cdot 2^{x+2} + 32 = 0$ के मूल ज्ञात कीजिए।

- (a) 2,3 (b) 1,3
(c) 1,2 (d) 5,2

15. If α, β are the roots of the equation $x^2 - 3px + p^2 = 0$ find p if $\alpha^2 + \beta^2 = \frac{7}{4}$

यदि α, β द्विघात समीकरण $x^2 - 3px + p^2 = 0$ के मूल हैं तथा $\alpha^2 + \beta^2 = \frac{7}{4}$ तो p का मान ज्ञात कीजिए।

- (a) $\pm \frac{2}{3}$ (b) $\pm \frac{1}{2}$
(c) $\frac{1}{2}$ (d) $\pm \frac{3}{2}$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
A	A	B	B	B	B	B	C	B	B
11	12	13	14	15					
B	A	B	A	B					

Worksheet solution

Sol 1

$$ax^2 - 5x + c = 0$$

$$d + \beta = \frac{-b}{a} = \frac{-(-5)}{a}$$

$$10 = \frac{5}{a} \quad (\because d + \beta = 10)$$

$$a = \frac{1}{2}$$

$$d \cdot \beta = \frac{c}{a} = 10$$

$$\frac{c}{a} = 10$$

$$c = 5$$

$$\therefore a = \frac{1}{2} \text{ \& } 5 \text{ Ans}$$

Sol 2

$$x^2 + px + q = 0$$

$$d + \beta = -p$$

$$d \cdot \beta = q$$

$$\frac{a}{\beta} + \frac{\beta}{d} = \frac{a^2 + \beta^2}{d\beta}$$

$$= \frac{(d + \beta)^2 - 2d\beta}{d\beta} = \frac{p^2 - 2q}{q} \text{ Ans}$$

Sol 3

$$x^2 + dx + \beta = 0$$

$$d + \beta = \frac{-b}{a} = -d$$

$$d\beta = \frac{c}{a} = \beta$$

$$\therefore d = 1$$

$$\beta = -2$$

$$d - \beta = 1 - (-2) \\ = 3 \text{ Ans}$$

Sol 4

$$\therefore d = \frac{1}{\beta} \Rightarrow d\beta = 1$$

$$px^2 + x + r = 0 \\ \therefore d\beta = \frac{r}{p} = 1$$

$$\therefore r = p \text{ Ans}$$

Sol 5

$$x^2 - (-5 - 3)x + (-5)(-3) = 0$$

$$x^2 + 8x + 15 = 0 \text{ Ans}$$

Sol 6 Roots are $5 + \sqrt{2}$

$5 - \sqrt{2}$

Sum = 10

Product = $(5 + \sqrt{2}) \cdot (5 - \sqrt{2})$
= 23

∴ Required equation is

$x^2 - 10x + 23 = 0$ Ans

Sol 7 $5x^2 - 7x + K = 0$

$C = a$ (∵ मूल 23-गुणित
के गुणक हैं)

∴ $K = 5$ Ans

Sol 8

$2x^2 - 5x + 7 = 0$

$d + \beta = \frac{5}{2}$

$d\beta = \frac{7}{2}$

∴ sum of ^{New} roots = $(2a + 3\beta)$
+ $(3a + 2\beta)$

= $5(d + \beta)$

= $5 \times \frac{5}{2} = \frac{25}{2}$

Product of New roots = $(2a + 3\beta)$
 $\times (3a + 2\beta)$

= $6(a^2 + \beta^2) + 13a\beta$

= $6[(a + \beta)^2 - 2a\beta] + 13a\beta$

= $6(d + \beta)^2 + a\beta$

= $6 \times \frac{25}{4} + \frac{7}{2} = 41$ Ans

∴ Required equation is:

$x^2 - \frac{25}{2}x + 41 = 0$

$2x^2 - 25x + 82 = 0$ Ans

Sol 9

$3x^2 - 20x + 17 = 0$

$x = \frac{1}{y}$ रखने पर-

$\frac{3}{y^2} - \frac{20}{y} + 17 = 0$

$3 - 20y + 17y^2 = 0$

सभी 12 ज -

$17x^2 - 20x + 3 = 0$ Ans

Sol 10

$x^2 - 2x + 4 = 0$

$\frac{a^2}{\beta^2}, \frac{\beta}{a^2}$

$d = \beta$ रखने पर

$\frac{a^3}{a^2}, \frac{\beta^3}{\beta^2} = a, \beta$

∴ समान समीकरण

= $x^2 - 2x + 4 = 0$ Ans

Sol 11

$$x^2 + x + 1 = 0$$

$$x^3 + x^2 + x = 0$$

$$x^3 - 1 = 0$$

$$x^3 = 1$$

$$\therefore \alpha^3, \beta^3$$

$$\alpha \cdot \alpha^3, \beta \cdot \beta^3$$

$$\alpha \cdot (\alpha^3)^1, \beta (\alpha^3)^1$$

$$\alpha, \beta$$

$$\therefore \text{Equation are } x^2 + x + 1 = 0$$

Sol 12

$$x = \frac{4}{3} \quad x = -\frac{3}{7}$$

$$\therefore \text{समीकरण } (3x-4)(7x+3)=0$$

Sol 13

$$3a, 2a$$

$$5a = -\frac{m}{12}$$

$$a = -\frac{m}{60}$$

$$3a \times 2a = \frac{5}{12}$$

$$6a^2 = \frac{5}{12}$$

$$\left(-\frac{m}{60}\right)^2 = \frac{5}{72}$$

$$m = 5\sqrt{10} \text{ Ans}$$

Sol 14

$$4^x - 3 \cdot 2^{x+2} + 32 = 0$$

$$2^{2x} - 3 \times 2^2 \times 2^x + 32 = 0$$

$$2^x = y$$

$$y^2 - 12y + 32 = 0$$

$$\wedge$$

$$y = 4, 8$$

$$2^x = 4 \quad \& \quad 2^x = 8$$

$$2^x = 2^2 \quad \& \quad 2^x = 2^3$$

$$x = 2, 3$$

$$\therefore 2, 3$$

Sol 15

$$x^2 - 3px + p^2 = 0$$

$$\alpha + \beta = 3p \quad \text{--- (i)}$$

$$\alpha\beta = p^2 \quad \text{--- (ii)}$$

$$\text{समीकरण (i) से}$$

$$(\alpha + \beta)^2 + 2\alpha\beta = 9p^2$$

$$\frac{7}{4} + 2p^2 = 9p^2$$

$$7p^2 = \frac{7}{4}$$

$$p^2 = \frac{1}{4}$$

$$p = \pm \frac{1}{2} \text{ Ans}$$