

SURDS AND INDICES

* **Indices** :- An Indices is a rational number with a rational power.

घातांक :- एक परिमेय संख्या जिसकी घात भी परिमेय होती है।

$$\text{Ex:- } 3^5, 10^{10}, 4^{-2} = \frac{1}{4^2}$$

* **Surds** :- An Irrational number with a power.

करणी :- एक अपरिमेय संख्या जिसकी कोई घात होती है।

$$\text{Ex:- } \sqrt{5}, \sqrt{3}, \sqrt{10}$$

$$\begin{array}{c} a^m \rightarrow \text{Power} \\ \downarrow \\ \text{base} \end{array}$$

LAW OF INDICES (घातांक के नियम)

$$1. (a^m)^n = a^{m \times n}$$

$$2. a^{m^n} = a^{m \times m \times m \times \dots \times n \text{ बार}}$$

$$3. a^m \times a^n = a^{m+n}$$

$$4. a^m \times b^m = (a \times b)^m$$

$$5. a^0 = 1$$

$$6. a^{-b} = \frac{1}{a^b} \Rightarrow \frac{1}{a^{-b}} = a^{+b}$$

$$7. \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$8. \frac{a^m}{a^n} = a^{m-n}$$

$$9. a^m = a^n \Rightarrow m = n$$

$$\text{Ex:- } 2^{3^{10}}$$

$$2^{3 \times 3 \times 3 \dots 10 \text{ बार}}$$

$$\text{Ex:- } 2^3 \times 2^4 = 2^7$$

$$\text{Ex:- } 2^4 \times 3^4 = (2 \times 3)^4 = 6^4$$

$$\text{Ex:- } 3^0 = 1, 1000^0 = 1$$

$$\text{Ex:- } 5^{-3} = \frac{1}{5^3}$$

$$\text{Ex:- } \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$$

$$\text{Ex:- } \frac{4^5}{4^3} = 4^{5-3} = 4^2$$

$$\text{Ex:- } 2^x = 2^3$$

$$(x=3)$$

Important formulae about indices:

घातांक से सम्बन्धित महत्वपूर्ण सूत्र:

If $a > 0, a \neq 1$, m and n are integers then

यदि $a > 0, a \neq 1$, तथा m और n पूर्णांक हो तो

(a) $a^m \times a^n = a^{m+n}$

(b) $a^m \times a^n \times a^p = a^{m+n+p}$

(c) $(a^m)^n = a^{mn}$

(d) $\frac{a^m}{a^n} = a^{m-n}$

(e) $a^0 = 1$

(f) $a^{-m} = \frac{1}{a^m}$

(g) $a^{m^n} = a^{(m^n)}$

(h) $(ab)^n = a^n b^n, (abc)^n = a^n b^n c^n$

Laws Of Surds (करणी के नियम)

1. $\sqrt[n]{a} = (a)^{\frac{1}{n}}$

Ex:- $\sqrt{2} = (2)^{\frac{1}{2}}$

$\sqrt[3]{3} = (3)^{\frac{1}{3}}$

2. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

$\sqrt[3]{5} = (5)^{\frac{1}{3}}$

$\sqrt[4]{7} = (7)^{\frac{1}{4}}$

Ex:- $\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

3. if, $a^x = b$
then $a = b^{\frac{1}{x}}$

Ex:- $x^3 = 216$

$x = (216)^{\frac{1}{3}}$

4. $a^x = b^y$
 $a = b^{\frac{y}{x}}$

5. $a^n = y$
 $a = y^{\frac{1}{n}}$

6. $a^x = b^y$
 $a^{\frac{1}{y}} = b^{\frac{1}{x}}$

Important formula for surd:

करणी से सम्बन्धित महत्वपूर्ण सूत्र:

(a) If (यदि) $a^n = y$ then (तो) $a = y^{1/n}$

(b) If (यदि) $a^x = b^y$ then (तो) $a = b^{y/x}$

(c) If (यदि) $a^x = b^y$ then (तो) $a^{1/y} = b^{1/x}$ etc.

(d) $x^n = a \Leftrightarrow x = \sqrt[n]{a}, (a \in R, a \geq 0)$

(e) If n is an odd positive integer and (यदि n एक विषम धन पूर्णांक है तथा) $a > 0$ then (तो) $\sqrt[n]{-a} = -\sqrt[n]{a}$ (here $m, n \geq 2$, and $a, b > 0$ then)

(f) $\sqrt[n]{a} = a^{\frac{1}{n}}$

(g) $(\sqrt[n]{a})^m = \sqrt[n]{a^m} = a^{\frac{m}{n}}$

(h) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

(i) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

(j) $\sqrt[n]{\sqrt[m]{a}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$

(k) $\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{\frac{1}{n}} \times a^{\frac{1}{m}} = a^{\frac{1}{n} + \frac{1}{m}} = a^{\frac{m+n}{mn}} = \sqrt[mn]{a^{m+n}}$

(l) $\frac{\sqrt[n]{a}}{\sqrt[m]{a}} = \frac{a^{\frac{1}{n}}}{a^{\frac{1}{m}}} = a^{\frac{1}{n} - \frac{1}{m}} = a^{\frac{m-n}{mn}} = \sqrt[mn]{a^{m-n}}$

TYPE-I

Q) $3^{\sqrt{x}} + 4^{\sqrt{x}} = 5^{\sqrt{x}}$ find x .

$$3^2 + 4^2 = 5^2$$

$\sqrt{x} = 2 \Rightarrow$ Square both side

$$x^{1/2} = 2$$

$$x = 2^2 = 4$$

$$x = 2^2 = 2^2 = 4$$

Q) $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$ find x

$$5^2 + 12^2 = 13^2$$

$$\sqrt{x} = 2$$

$$x = 2^2 = 4$$

Q) If $3^{x+y} = 81$ and $81^{x-y} = 3$ then the value of $\frac{x}{y}$ is

$$3^{x+y} = 3^4$$

$$\boxed{x+y=4}$$

$$(3^4)^{(x-y)} = 3^1$$

$$3^{4(x-y)} = 3^1$$

$$4(x-y) = 1$$

$$\boxed{x-y = \frac{1}{4}}$$

$$a^m = a^n$$

$$\boxed{m=n}$$

$$\frac{x+y}{x-y} = \frac{4}{\frac{1}{4}} = \frac{16}{1}$$

C & D Rule $\frac{x}{y} = \frac{16+1}{16-1} = \frac{17}{15}$

Q) If $3^{x-1} + 3^{3-x} = 6$, then what is $2^{x-1} + 2^{3-x}$ equal

$$\frac{3^x}{3^1} + \frac{3^3}{3^x} = 6$$

$$\frac{3^{2x} + 3^4}{3 \times 3^x} = 6$$

$x=2$

$$\begin{array}{rcl} 3^{2x} + 81 & = & 18 \times 3^x \\ 3^4 + 81 & & 18 \times 3^2 \\ 81 + 81 & & 18 \times 9 \\ 162 & = & 162 \end{array}$$

$$2^{x-1} + 2^{3-x}$$

$$2^{2-1} + 2^{3-2}$$

$$2^1 + 2^1$$

$$4$$

① $a^{m-n} = \frac{a^m}{a^n}$

② $a^{m+n} = a^m \times a^n$

Q) If $5^{x+1} - 5^{x-1} = 600$, then what is the value of 10^{2x} ?

$$5^1 \times 5^x - \frac{5^x}{5^1}$$

$$5^x \left(5 - \frac{1}{5} \right) = 600$$

$$5^x \left(\frac{24}{5} \right) = 600 \Rightarrow 25$$

$$5^x = 125 = 5^3$$

$$\boxed{x=3}$$

$$10^{2x}$$

$$10^{2 \times 3} = 10^6$$

$$1000000$$

Q) If $3^{x+3} \times 9^{2x-5} = 3^{3x+7}$ then the value of x is :

$$3^{x+3} \times (3^2)^{2x-5}$$

$$3^{x+3} \times 3^{4x-10}$$

$$3^{5x-7} = 3^{3x+7}$$

$$5x-7 = 3x+7$$

$$2x = 14$$

$$x = 7$$

Q) If $\frac{4^x \times 16^8 \times 8^{\frac{2}{3}}}{2^{3x+2} \times (256)^3} = 32^{\frac{x}{2}+2}$ then the value of x is

$$\frac{2^{2x} \times (2^4)^8 \times (2^{\frac{2}{3}})^{\frac{2}{3}}}{2^{3x+2} \times (2^8)^3} = (2^5)^{\frac{x}{2}+2}$$

$$\frac{2^{2x} \times 2^{32} \times 2^2}{2^{3x+2} \times 2^{24}} = 2^{\frac{5x}{2}+10}$$

$$\frac{2^{2x} \times 2^8 \times 2^2}{2^{3x+2}} = \frac{2^{2x+10}}{2^{3x+2}}$$

$$2^{2x+10-3x-2}$$

$$= 2^{8-x}$$

$$2^{8-x} = 2^{\frac{5x}{2}+10}$$

$$8-x = \frac{5x}{2}+10$$

$$-2 = \frac{5x}{2}+x$$

$$-2 = \frac{7x}{2}$$

$$x = -\frac{4}{7}$$

Q) If $\sqrt{6}^5 \times (36)^2 = \sqrt[3]{6} \times 6^{\frac{x}{2}} \times 6\sqrt{6}$ then find the value

$$(6^{\frac{1}{2}})^5 \times (6^2)^2 = 6^{\frac{1}{3}} \times 6^{\frac{x}{2}} \times 6^1 \times 6^{\frac{1}{2}}$$

$$6^{\frac{5}{2}} \times 6^4 = 6^{\frac{1}{3} + \frac{x}{2} + 1 + \frac{1}{2}}$$

$$\frac{3}{2} + \frac{1}{3} = \frac{11}{6} + \frac{x}{2}$$

$$6^{\frac{13}{2}} = 6^{\frac{22+6x}{12}}$$

$$\frac{22+6x}{12}$$

$$\frac{13}{2} = \frac{22+6x}{12}$$

$$22+6x=78$$

$$6x=56$$

$$x = \frac{56}{6} = \frac{28}{3}$$

$$\frac{28}{3}$$

1. $3^{\sqrt{x}} + 4^{\sqrt{x}} = 5^{\sqrt{x}}$ find x .

- (a) 2
- (b) 3
- (c) 4
- (d) 5

2. $3^x - 3^{x-1} = 486$. Find x

- (a) 7
- (b) 9
- (c) 5
- (d) 6

3. If $2^{n-1} + 2^{n+1} = 320$ then the value of n is

यदि $2^{n-1} + 2^{n+1} = 320$ तो n का मान है

- (a) 6
- (b) 8
- (c) 5
- (d) 7

4. $9^{(5x-2)} = 27^4$ find $x = ?$

- (a) 2
- (b) 3.2
- (c) 4
- (d) 1.6

5. If $27^{2n-1} = (243)^3$ then the value of n is :

यदि $27^{2n-1} = (243)^3$ तो n का मान है :

- (a) 3
- (b) 6
- (c) 7
- (d) 9

6. If $3^{x+y} = 81$ and $81^{x-y} = 3$ then the value of $\frac{x}{y}$ is

यदि $3^{x+y} = 81$ तथा $81^{x-y} = 3$ तो $\frac{x}{y}$ का मान क्या होगा?

- | | |
|---------------------|---------------------|
| (a) $\frac{15}{17}$ | (b) $\frac{17}{30}$ |
| (c) $\frac{15}{34}$ | (d) $\frac{17}{15}$ |

7. If $11\sqrt{n} = \sqrt{112} + \sqrt{343}$ then the value of n is:

यदि $11\sqrt{n} = \sqrt{112} + \sqrt{343}$ तो n का मान है

- (a) 3
- (b) 13
- (c) 11
- (d) 7

8. simplify $\frac{2^{3a} \times 3^{2a} \times 5^a \times 6^a}{8^a \times 9^{3a/2} \times 10^a}$

(a) 5

(b) 3

(c) 0

(d) 1

9. $\frac{18^4 \times 27^3 \times 36^5}{(1024) \times (243)^4} = (2^{1/2})^8 \times 3^{x/2}$, find the value of x

$\frac{18^4 \times 27^3 \times 36^5}{(1024) \times (243)^4} = (2^{1/2})^8 \times 3^{x/2}$, x का मान ज्ञात करें

(a) 14

(b) 21

(c) 7

(d) 16

10. The value of $\frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$ is

$\frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$ का मान है

(a) 3

(d) 9

(c) 6

(d) 12

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
C	D	D	D	A	D	D	D	A	B

Worksheet solution

Sol 1

$$3^{\sqrt{n}} + 4^{\sqrt{n}} = 5^{\sqrt{n}}$$

we know

$$3^2 + 4^2 = 5^2$$

$$\therefore \sqrt{n} = 2$$

$$n = 4$$

Sol 2

$$3^n - 3^{n-1} = 486$$

$$3^n \left[1 - \frac{1}{3} \right] = 486$$

$$3^n \left[\frac{2}{3} \right] = 486$$

$$3^n = \frac{486 \times 3}{2}$$

$$3^n = 3^6$$

$$\therefore n = 6$$

Sol 3

$$2^{n-1} + 2^{n+1} = 320$$

$$2^n \left[\frac{1}{2} + 2 \right] = 320$$

$$2^n = \frac{320 \times 2}{5}$$

$$2^n = 2^7$$

$$\therefore n = 7$$

Sol 4

$$9^{5n-2} = 27^4$$

$$3^{10n-4} = 3^{12} \left[27^4 = (3^3)^4 \right]$$

$$10n - 4 = 12$$

$$10n = 16$$

$$n = 1.6$$

Sol 5

$$27^{2n-1} = (243)^3$$

$$(3^3)^{2n-1} = (3^5)^3$$

$$3^{6n-3} = 3^{15}$$

$$6n - 3 = 15$$

$$6n = 18$$

$$n = 3$$

Sol 6

$$3^{n+y} = 81 \text{ \& } 81^{n-y} = 3$$

$$3^{n+y} = 3^4, \quad n+y = 4$$

$$\& (3^4)^{n-y} = 3^1, \quad 4n-4y = 1$$

$$4n + 4y = 16$$

$$4n - 4y = 1$$

$$8n = 17$$

and

$$8y = 15$$

$$\therefore \frac{n}{y} = \frac{17}{15}$$

Sol-7

$$11\sqrt{n} = \sqrt{112} + \sqrt{343}$$

$$11\sqrt{n} = 4\sqrt{7} + 7\sqrt{7}$$

$$11\sqrt{n} = 11\sqrt{7}$$

$$\therefore n = 7$$

Sol 8

$$\frac{2^{3a} \times 3^{2a} \times 5^a \times 6^a}{8^a \times 9^{3a/2} \times 10^a}$$

$$\frac{2^{3a} \times 3^{2a} \times 30^a}{2^{3a} \times (3^2)^{3a/2} \times 10^a}$$

$$\frac{2^{3a} \times 3^{2a} \times 30^a}{2^{3a} \times 3^{3a} \times 10^a}$$

$$3^{-a} \times 3^a = 1$$

Sol 9

$$\frac{18^4 \times 27^3 \times 36^5}{(1024) \times (243)} = (2^{1/2})^8 \times 3^{7/2}$$

$$\frac{(2 \times 3^2)^4 \times (3^3)^3 \times (2^2 \times 3^2)^5}{2^{10} \times (3^5)^4} = 2^4 \times 3^{\frac{7}{2}}$$

$$\frac{2^4 \times 2^{10} \times 3^9 \times 3^9 \times 3^{10}}{2^{10} \times 3^{20}} = 2^4 \times 3^{\frac{7}{2}}$$

$$2^4 \times 3^7 = 2^4 \times 3^{\frac{7}{2}}$$

$$3^7 = 3^{\frac{7}{2}}$$

$$7 = \frac{7}{2}$$

$$n = 14$$

Sol 10

$$\Rightarrow \frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$$

$$= \frac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$$

$$= \frac{3^n \times 3^{2n+1}}{3^2 \times 3^{n-1}}$$

$$= \frac{3^{3n} \times 3}{3^{3n} \times 3^{-1}} = 9$$

SURDS AND INDICES

Q) If $\frac{4^n \times 4^8 - 4^{(n+1)}}{4 \times 4^{n+7}} = x$, then find $x \times 4^7$?

$$\frac{4^n \times 4^8 - 4^1 \times 4^n}{4 \times 4^7 \times 4^n} = x$$

$$\frac{4^n (4^8 - 4^1)}{4 \times 4^n} = x \times 4^7$$

$$\frac{4^1 (4^7 - 1)}{4} = x \times 4^7$$

$$(4^7 - 1)$$

Q) Given $\sqrt{2} = 1.414$. The value of $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$ is

$$\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$$

$$\sqrt{4 \times 2} \quad \sqrt{16 \times 2} \quad \sqrt{64 \times 2} \quad \sqrt{25 \times 2}$$

$$2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2}$$

$$\sqrt{2} (2 + 8 - 24 + 20)$$

$$= 6\sqrt{2}$$

$$6 \times 1.414 = 8.484$$

Q) If a, b are rational number and $a\sqrt{2} + b\sqrt{3} = \sqrt{98} + \sqrt{108} - \sqrt{48} - \sqrt{72}$ then the values of a, b are respectively.

यदि a, b परिमेय संख्याएँ हैं और $a\sqrt{2} + b\sqrt{3} = \sqrt{98} + \sqrt{108} - \sqrt{48} - \sqrt{72}$ है तो a, b के मान क्रमशः हैं।

$$\sqrt{98} + \sqrt{108} - \sqrt{48} - \sqrt{72}$$

$$\sqrt{49 \times 2} \quad \sqrt{36 \times 3} \quad \sqrt{16 \times 3} \quad \sqrt{36 \times 2}$$

$$7\sqrt{2} + 6\sqrt{3} - 4\sqrt{3} - 6\sqrt{2}$$

$$1\sqrt{2} + 2\sqrt{3} = a\sqrt{2} + b\sqrt{3}$$

On comparing $a=1, b=2$

$$\# \left(\left((a^m)^n \right)^o \right)^p = a^{m \times n \times o \times p}$$

Q) $\left[\left\{ \left(-\frac{1}{2} \right)^2 \right\}^{-2} \right]^{-1}$ is equal to / के बराबर है ।

$$\left(-\frac{1}{2} \right)^{2 \times (-2) \times (-1)}$$

$$\left(-\frac{1}{2} \right)^4 = +\frac{1}{16}$$

$$(-a)^{\text{odd}} = -(a)^{\text{odd}}$$

$$(-a)^{\text{even}} = +(a)^{\text{even}}$$

Q) The value of $\sqrt[3]{0.000125}$ is
 $\sqrt[3]{0.000125}$ का मान है

$$\left(\frac{125}{1000000} \right)^{1/3}$$

$$\left[\frac{5^3}{100^3} \right]^{1/3}$$

$$\left[\left(\frac{5}{100} \right)^3 \right]^{1/3} = \left(\frac{5}{100} \right)^{3 \times \frac{1}{3}}$$

$$\frac{5}{100} = 0.05$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b} \right)^n$$

$$(a^m)^n = a^{m \times n}$$

Q) $\sqrt{\sqrt[3]{0.004096}}$ is equal to.
 $\sqrt{\sqrt[3]{0.004096}}$ का मान है

$$\left[\left[\frac{4096}{10000000} \right]^{1/3} \right]^{1/2}$$

$$\left[\frac{4096}{10000000} \right]^{1/6}$$

$$\left(\frac{4^6}{10^6} \right)^{1/6} = \left[\left(\frac{4}{10} \right)^6 \right]^{1/6}$$

$$\frac{4}{10} = 0.4$$

Q) Find the value of $\sqrt{625} + \sqrt{\frac{625}{100}} + \sqrt{\frac{625}{10000}} + \sqrt{\frac{625}{1000000}}$

$\sqrt{625} + \sqrt{\frac{625}{100}} + \sqrt{\frac{625}{10000}} + \sqrt{\frac{625}{1000000}}$ का मान ज्ञात कीजिए ।

$$25 + \frac{25}{10} + \frac{25}{100} + \frac{25}{1000}$$

$$25 + 2.5 + 0.25 + 0.025$$

$$27.775$$

Q) Which of the following statement (s) is / are true

निम्नलिखित में से कौन - सा कथन सत्य है ।

I. $\sqrt{64} + \sqrt{0.0064} + \sqrt{0.81} + \sqrt{0.0081} = 9.07$

II. $\sqrt{0.0010201} + \sqrt{98.01} + \sqrt{0.25} = 11.51$

a Only I / केवल I

b Only II / केवल II

c Both I and II / I और II दोनों

d Neither I nor II / न तो I न ही II

I. $\sqrt{64} + \sqrt{0.0064} + \sqrt{0.81} + \sqrt{0.0081} = 9.07$

$$8 + 0.08 + 0.9 + 0.09$$

$$\begin{array}{r} 0.08 \\ 0.9 \\ 0.09 \\ 8 + 1.07 \\ = 9.07 \end{array}$$

Only I

Q) $6\sqrt{(27)^{-\frac{2}{3}} + (8)^{-\frac{2}{3}}}$ Simplify value of / का सरलीकरण मान है ।

$$(3^3)^{-\frac{2}{3}} + (2^3)^{-\frac{2}{3}}$$

$$6\sqrt{3^{-2} + 2^{-2}}$$

$$6\sqrt{\frac{1}{3^2} + \frac{1}{2^2}}$$

$$6\sqrt{\frac{1}{9} + \frac{1}{4}}$$

$$a^{-m} = \frac{1}{a^m}$$

$$\sqrt[6]{\frac{13}{36}} = \sqrt[6]{\frac{13}{6 \times 6}} = \sqrt[6]{13} \times \sqrt[6]{\frac{1}{6 \times 6}} = \sqrt[6]{13}$$

Q) If $\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$ then x is equal to

यदि $\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$ तो x किसके बराबर होगा।

$$1 + \frac{x}{144} = \frac{169}{144}$$

$$1 + \frac{x}{144} = 1 + \frac{25}{144}$$

$$x = 25$$

Q) On comparing the following two numeric expressions we find that ?
निम्नलिखित दो संख्यात्मक व्यंजक की तुलना करने पर हम पाते हैं कि ?

$$\left[\left(2\frac{1}{9} \right)^{2\frac{1}{2}} \right]^{\frac{3}{5}} \text{ and / और } \left[\left(1\frac{2}{3} \right)^5 \right]^{\frac{3}{5}}$$

$$\left[\left(2\frac{1}{9} \right)^{2\frac{1}{2}} \right]^{\frac{3}{5}}$$

$$\left(\frac{25}{9} \right)^{\frac{8}{2} \times \frac{3}{5}}$$

$$\left[\left(\frac{5}{3} \right)^2 \right]^{\frac{3}{2}}$$

$$\frac{125}{27}$$

$$\left[\left(1\frac{2}{3} \right)^5 \right]^{\frac{3}{5}}$$

$$\left(\frac{5}{3} \right)^{5 \times \frac{3}{5}}$$

$$\frac{125}{27}$$

Both the expression are equal
दोनों व्यंजक समान हैं।

TYPE-II

- Q) Which one of the following is the least?
निम्नलिखित में से कौन सा सबसे छोटा है ?

$$\sqrt{3}, \sqrt[3]{2}, \sqrt{2} \text{ and } \sqrt[3]{4}$$

$(3)^{\frac{1}{2} \times 6^3}$	$(2)^{\frac{1}{3} \times 6^2}$	$(2)^{\frac{1}{2} \times 6^3}$	$(4)^{\frac{1}{3} \times 6^2}$	$\text{LCM}(2,3) = 6$
3^3	2^2	2^3	4^2	
27	4	8	16	
	$\hookrightarrow \sqrt[3]{2}$			

Ascending order आरोही क्रम

$$\sqrt[3]{2} < \sqrt{2} < \sqrt[3]{4} < \sqrt{3}$$

Descending order अवरोही क्रम

$$\sqrt{3} > \sqrt[3]{4} > \sqrt{2} > \sqrt[3]{2}$$

- Q) Which one of the following is the biggest?
निम्नलिखित में से कौन सा सबसे बड़ा है

$$\sqrt[3]{4}, \sqrt[4]{6}, \sqrt[6]{15} \text{ and } \sqrt[12]{245}$$

$(4)^{\frac{1}{3} \times 12^4}$	$(6)^{\frac{1}{4} \times 12^3}$	$(15)^{\frac{1}{6} \times 12^2}$	$(245)^{\frac{1}{12} \times 12^1}$	$\text{LCM}(3,4,6,12) = 12$
4^4	6^3	15^2	245^1	
256	216	225	245	
$\hookrightarrow \sqrt[3]{4}$				

- Q) Arrange the following in ascending order
निम्नलिखित को आरोही क्रम में व्यवस्थित करें

$$\sqrt[3]{9}, \sqrt[4]{20}, \sqrt[6]{25}$$

$9^{\frac{1}{3} \times 12^4}$	$20^{\frac{1}{4} \times 12^3}$	$25^{\frac{1}{6} \times 12^2}$	$\text{LCM}(3,4,6) = 12$
9^4	20^3	25^2	
$9^2 \times 9^2$	$\downarrow$	$\downarrow$	
81 × 81	8000	625	
80 × 80			
6400			

$$\sqrt[6]{25} < \sqrt[3]{9} < \sqrt[4]{20}$$

Q) The greatest of the numbers
 $(2.89)^{0.5}$, $2 - (0.5)^2$, $1 + \frac{0.5}{1 - \frac{1}{2}}$, $\sqrt{3}$

सबसे बड़ी संख्या $(2.89)^{0.5}$, $2 - (0.5)^2$, $1 + \frac{0.5}{1 - \frac{1}{2}}$, $\sqrt{3}$ है।

$$[(1.7)^2]^{\frac{1}{2}}, (2 - \frac{1}{4}), 1 + \frac{0.5}{\frac{1}{2}}, 1.73$$

$$1.7, \frac{7}{4}, 1+1, 1.73$$

$$1.7, 1.75, 2, 1.73$$

$$\frac{1+0.5}{1-\frac{1}{2}}$$

Q) The smallest among the numbers is
 संख्याओं में सबसे छोटी है।

$$2^{250}, 3^{150}, 5^{100} \text{ and } 4^{200}$$

$$\text{HCF}(250, 150, 100, 200)$$

$$\begin{array}{cccc} \textcircled{50} (5, 3, 2, 4) & & & \\ \frac{250}{50} & , & \frac{150}{50} & , & \frac{100}{50} & , & \frac{200}{50} \\ 2^5 & & 3^3 & & 5^2 & & 4^4 \\ 32 & & 27 & & 25 & & 256 \\ & & & & \textcircled{5^{100}} & & \end{array}$$

Q) The greatest number among 2^{60} , 3^{48} , 4^{36} and 5^{24}
 2^{60} , 3^{48} , 4^{36} और 5^{24} और में से सबसे बड़ी संख्या है।

$$\text{HCF}(60, 48, 36, 24)$$

$$\textcircled{12} (5, 4, 3, 2)$$

$$\begin{array}{cccc} \frac{60}{12} & , & \frac{48}{12} & , & \frac{36}{12} & , & \frac{24}{12} \\ 2^5 & & 3^4 & & 4^3 & & 5^2 \\ 32 & & 81 & & 64 & & 25 \\ & & \textcircled{3^{48}} & & & & \end{array}$$

Q) $2^{60}, 3^{70}, 4^{50}, 5^{20}$ find the value of smallest^{and} largest number.

$2^{60}, 3^{70}, 4^{50}, 5^{20}$ सबसे छोटी और सबसे बड़ी संख्या का मान ज्ञात कीजिए।

$$\text{HCF}(60, 70, 50, 20) = 10$$

$$\begin{array}{cccc} 2^6 & 3^7 & 4^5 & 5^2 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 64 & 2187 & 1024 & 25 \end{array}$$

$$\text{Smallest} = 5^{20}$$

$$\text{Largest} = 3^{70}$$

$$(\sqrt{3} - \sqrt{2}) \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$\frac{3-2}{\sqrt{3}+\sqrt{2}} = \frac{1}{\sqrt{3}+\sqrt{2}} \quad \begin{array}{l} \text{बड़ी} \\ \text{छोटी} \end{array}$$

$$\begin{array}{cc} 0.5 & 0.33 \\ \frac{1}{2} & \frac{1}{3} \\ \downarrow & \downarrow \\ \text{बड़ी} & \text{छोटी} \end{array}$$

- Reciprocal में अगर Numerator Same है तो जिसका Denominator छोटा होगा वो संख्या बड़ी होगी और जिसका Denominator बड़ा होगा वो संख्या छोटी होगी।

Q) Which is the greatest among इनमें से सबसे बड़ा कौन है ?

$$(\sqrt{19} - \sqrt{17}), (\sqrt{13} - \sqrt{11}), (\sqrt{7} - \sqrt{5}) \text{ and } (\sqrt{5} - \sqrt{3})$$

$$\begin{array}{ccccccc} \text{सबसे छोटी} & \left(\frac{2}{\sqrt{19}+\sqrt{17}} \right) & \frac{2}{\sqrt{13}+\sqrt{11}} & \frac{2}{\sqrt{7}+\sqrt{5}} & \left(\frac{2}{\sqrt{5}+\sqrt{3}} \right) & \text{सबसे बड़ी} \\ & \underline{36} & \underline{24} & \underline{12} & \underline{8} & \end{array}$$

$$\sqrt{5} - \sqrt{3}$$

- Minus (-) का Sign है और diff. same है तो जिसका योग सबसे छोटा होगा वो संख्या सबसे बड़ी होगी।
- अगर (+) plus का Sign है तो जिसका योग सबसे बड़ा वो संख्या सबसे बड़ी।

1. If $\sqrt{6} = 2.45$, then find the value of $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$.

यदि $\sqrt{6} = 2.45$ है, तो $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ का मान ज्ञात कीजिए।

- (a) 7.7
- (b) 9.9
- (c) 6.6
- (d) 8.78

2. The value of $\sqrt{513 - \sqrt{144} - \sqrt{81} - \sqrt{64}}$ का मान क्या है?

- (a) 22
- (b) 21
- (c) 28
- (d) 19

3. Solve/ज्ञात कीजिए:

$$\sqrt{(261 - \sqrt{(35 - \sqrt{(95 + \sqrt{25}))})})}$$

- (a) 14
- (b) 15
- (c) 16
- (d) 24

4. If $\left[\left\{ \left(\frac{2}{3} \right)^3 \right\}^{(2x+3)} \right]^{\frac{-3}{4}} = \left[\left\{ \left(\frac{2}{3} \right)^{\frac{2}{3}} \right\}^{3x+7} \right]^{\frac{-6}{5}}$, then find the value of $\sqrt{2 - 42x}$

यदि $\left[\left\{ \left(\frac{2}{3} \right)^3 \right\}^{(2x+3)} \right]^{\frac{-3}{4}} = \left[\left\{ \left(\frac{2}{3} \right)^{\frac{2}{3}} \right\}^{3x+7} \right]^{\frac{-6}{5}}$ है, तो $\sqrt{2 - 42x}$ का मान ज्ञात

कीजिए।

- (a) 5
- (b) 6
- (c) 3
- (d) 4

5. The value of $\sqrt[4]{\sqrt{100000000}} - \sqrt[3]{9261}$

$\sqrt[4]{\sqrt[4]{1000000000}} - \sqrt[3]{9261}$ का मान ज्ञात करें।

- (a) -14
- (b) -11
- (c) -51
- (d) -19

6. The value of $[(\sqrt{529}) + (\sqrt{5.29}) + (\sqrt{0.0529})]$ is:

$[(\sqrt{529}) + (\sqrt{5.29}) + (\sqrt{0.0529})]$ का मान क्या है?

- (a) 25.323
- (b) 25.53
- (c) 23.253
- (d) 23.53

7. $\left[8 - \left(\frac{4^{\frac{9}{4}} \sqrt{2.2^2}}{2\sqrt{2^{-2}}} \right)^{\frac{1}{2}} \right]$ is equal to

- (a) 32
- (b) 8
- (c) 1
- (d) 0

8. $\sqrt{\frac{(0.1)^2 + (0.01)^2 + (0.009)^2}{(0.01)^2 + (0.001)^2 + (0.0009)^2}} = ?$

- (a) 10^2
- (b) 10
- (c) 0.1
- (d) 0.01

9. Which is greatest number among $\sqrt[3]{8}$, $\sqrt[2]{4}$, $\sqrt[6]{64}$?

$\sqrt[3]{8}$, $\sqrt[2]{4}$, $\sqrt[6]{64}$ में से सबसे बड़ी संख्या कौन-सी है?

- (a) $\sqrt[3]{8}$
- (b) $\sqrt[2]{4}$
- (c) $\sqrt[6]{64}$
- (d) सभी बराबर हैं

10. $\sqrt[4]{3}$, $\sqrt[5]{4}$, $\sqrt[10]{12}$, 1 What is the smallest number in

$\sqrt[4]{3}, \sqrt[5]{4}, \sqrt[10]{12}, 1$ में सबसे छोटी संख्या कौन-सी है?

- (a) 1
- (b) $\sqrt[4]{3}$
- (c) $\sqrt[5]{4}$
- (d) $\sqrt[10]{12}$

11. $\sqrt{5} - \sqrt{3}, \sqrt{7} - \sqrt{5}; \sqrt{9} - \sqrt{7}, \sqrt{11} - \sqrt{9}$ largest number is

$\sqrt{5} - \sqrt{3}, \sqrt{7} - \sqrt{5}; \sqrt{9} - \sqrt{7}, \sqrt{11} - \sqrt{9}$ सबसे बड़ी संख्या है

- (a) $\sqrt{5} - \sqrt{3}$
- (b) $\sqrt{7} - \sqrt{5}$
- (c) $\sqrt{9} - \sqrt{7}$
- (d) $\sqrt{11} - \sqrt{9}$

12. $(\sqrt{8} + \sqrt{5}), (\sqrt{7} + \sqrt{6}), (\sqrt{10} + \sqrt{3}), (\sqrt{11} + \sqrt{12})$ who is smallest number

$(\sqrt{8} + \sqrt{5}), (\sqrt{7} + \sqrt{6}), (\sqrt{10} + \sqrt{3}), (\sqrt{11} + \sqrt{12})$ सबसे छोटी संख्या कौन है?

- (a) $\sqrt{8} + \sqrt{5}$
- (b) $\sqrt{7} + \sqrt{6}$
- (c) $\sqrt{10} + \sqrt{3}$
- (d) $\sqrt{11} + \sqrt{12}$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10	11	12
B	A	C	A	B	B	D	B	D	A	A	C

Worksheet solution

Sol 1

$$= \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$= \frac{5 + 2\sqrt{6}}{1}$$

$$= 5 + 4.9 = 9.9$$

Sol 2

$$\sqrt{513 - \sqrt{144} - \sqrt{81} - \sqrt{64}}$$

$$\sqrt{513 - 12 - 9 - 8}$$

$$\sqrt{484} = 22$$

Sol 3

$$\sqrt{\left(261 - \sqrt{\left(35 - \sqrt{\left(95 + \sqrt{25}\right)}\right)}\right)}$$

$$= \sqrt{\left(261 - \sqrt{\left(35 - \sqrt{100}\right)}\right)}$$

$$= \sqrt{\left(261 - \sqrt{25}\right)}$$

$$= \sqrt{261 - 5} = \sqrt{256}$$

$$= 16$$

Sol 4

$$\left[\left\{\left(\frac{2}{3}\right)\right\}^{2n+3}\right]^{\frac{-3}{4}} = \left[\left\{\left(\frac{2}{3}\right)\right\}^{\frac{2}{3} \times 3n+7}\right]^{\frac{-6}{5}}$$

$$\left[\left(\frac{2}{3}\right)^{6n+9}\right]^{\frac{3}{4}} = \left[\left(\frac{2}{3}\right)^{2n+\frac{2}{3} \times 7}\right]^{\frac{6}{5}}$$

$$\left(2n + \frac{14}{3}\right) \frac{6}{5} = (6n+9) \times \frac{3}{4}$$

$$\frac{12n}{5} + \frac{28}{5} = \frac{9n}{2} + \frac{27}{4}$$

$$\frac{12n}{5} - \frac{9n}{2} = \frac{27}{4} - \frac{28}{5}$$

$$\frac{24n - 45n}{10} = \frac{135 - 112}{20}$$

$$-42n = 23$$

$$n = -\frac{23}{42}$$

$$\therefore \sqrt{2 - 42 \times \frac{-23}{42}}$$

$$= \sqrt{2 + 23} = 5$$

Sol 5

$$= \sqrt[4]{\sqrt{10000000000} - \sqrt[3]{9261}}$$

$$= \sqrt{100} - 21$$

$$= 10 - 21 = -11$$

Sol 6

$$[(\sqrt{529} + (\sqrt{5.29}) + (\sqrt{0.0529}))]$$

$$= 23 + 2 \cdot 3 + 0.23$$

Sol 7

$$8 - \left[\frac{4^{\frac{9}{4}} \sqrt{2 \cdot 2^2}}{2 \sqrt{2^{-2}}} \right]^{\frac{1}{2}}$$

$$= 8 - \left[\frac{2^{\frac{9}{2}} \times 2^{\frac{3}{2}}}{1} \right]^{\frac{1}{2}}$$

$$= 8 - \left[\frac{2^6}{1} \right]^{\frac{1}{2}}$$

$$= 8 - 8 = 0$$

Sol 8

$$\sqrt{\frac{(0.1)^2 + (0.01)^2 + (0.009)^2}{(0.01)^2 + (0.001)^2 + (0.009)^2}}$$

$$= \sqrt{\frac{100 \times [(0.1)^2 + (0.01)^2 + (0.009)^2]}{(0.1)^2 + (0.01)^2 + (0.009)^2}}$$

$$= \sqrt{100} = 10$$

Sol 9

$$= 3\sqrt{8}, 2\sqrt{4}, 6\sqrt{64}$$

$$= 8^{\frac{1}{2}}, 4^{\frac{1}{2}}, 64^{\frac{1}{6}}$$

$$= 2, 2, 2$$

अतः सभी संशय हैं।

Sol 10

$$4\sqrt{3}, 5\sqrt{4}, 10\sqrt{12}, 1$$

$$= 3^{\frac{1}{4} \times 20}, 4^{\frac{1}{5} \times 20}, 12^{\frac{1}{10} \times 20}, 1$$

$$= 3^5, 4^4, 12^2, 1$$

Sol 11

$$(\sqrt{5} - \sqrt{3}), (\sqrt{7} - \sqrt{5})$$

$$(\sqrt{9} - \sqrt{7}), (\sqrt{11} - \sqrt{9})$$

$$(\sqrt{5} - \sqrt{3})_{\text{Ans}}$$

Note - जहाँ अंश समान हो, वहाँ अंश छोटी संख्या ही छोटी मानेगी।

Sol 12

$$(\sqrt{8} + \sqrt{15})^2 = 13 + 2\sqrt{40}$$

$$(\sqrt{7} + \sqrt{6})^2 = 13 + 2\sqrt{42}$$

$$(\sqrt{10} + \sqrt{3})^2 = 13 + 2\sqrt{30}$$

$$(\sqrt{11} + \sqrt{12})^2 = 23 + 2\sqrt{132}$$



SURDS AND INDICES

Q) $(\sqrt{5} + \sqrt{4})$, $(\sqrt{7} + \sqrt{2})$, $(\sqrt{6} + \sqrt{3})$ find the value of largest and smallest number.

$(\sqrt{5} + \sqrt{4})$, $(\sqrt{7} + \sqrt{2})$, $\sqrt{6} + \sqrt{3}$ सबसे बड़ी और सबसे छोटी संख्या का मान ज्ञात करें।

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(\sqrt{5} + \sqrt{4})^2 = 9 + 2\sqrt{20} \text{ --- ①}$$

$$(\sqrt{7} + \sqrt{2})^2 = 9 + 2\sqrt{14} \text{ --- ③}$$

$$(\sqrt{6} + \sqrt{3})^2 = 9 + 2\sqrt{18} \text{ --- ②}$$

largest: $(\sqrt{5} + \sqrt{4})$

Smallest: $(\sqrt{7} + \sqrt{2})$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

* $\sqrt{a} + \sqrt{b}$, $\sqrt{c} + \sqrt{d}$, $\sqrt{e} + \sqrt{f}$

where $(a+b) = (c+d) = (e+f)$

then जिस surd के root के अन्दर वाली संख्याओं का Multiplication बड़ा होगा वो surd बड़ा हो जाएगा।

TYPE-III

$$\bullet (a+b)^2 = a^2 + b^2 + 2ab \longrightarrow (7+3)^2 = 58 + 2 \times 21$$

$$\bullet (a-b)^2 = a^2 + b^2 - 2ab$$

$$\bullet (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$\bullet (a+b)^2 - (a-b)^2 = 4ab$$

$$\bullet (\sqrt{a})^2 = a$$

$$\begin{array}{l} a^2 + b^2 \quad 2 \quad a \times b \\ 7^2 + 3^2 = 58 \end{array}$$

Q) What is the square root of $16 + 6\sqrt{7}$?

$16 + 6\sqrt{7}$ का वर्गमूल क्या है?

$$\begin{aligned} & \sqrt{16 + 6\sqrt{7}} \\ & \downarrow \\ & (a^2 + b^2) + 2 \times \underbrace{3\sqrt{7}}_{\substack{3 \times \sqrt{7} \\ 9 + 7 = 16}} - (ab) \\ & \sqrt{(3 + \sqrt{7})^2} \\ & = 3 + \sqrt{7} \end{aligned}$$

Q) What is the square root of $15 - 4\sqrt{14}$?
 $15 - 4\sqrt{14}$ का वर्गमूल क्या है ?

$$\begin{aligned} & \sqrt{15 - 4\sqrt{14}} \\ & a^2 + b^2 - 2 \times \underbrace{2\sqrt{14}}_{a \times b} = \frac{\sqrt{2}}{\sqrt{7}} \\ & = 2\sqrt{2} - \sqrt{7} \\ & 8 + 7 = 15 \end{aligned}$$

Q) If $x = \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$ then find the value of x . Where $x > 0$

यदि $x = \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$ तो x का मान ज्ञात करें जहाँ $x > 0$

$$\begin{aligned} & \begin{array}{c} 2 \times 2\sqrt{3} \\ \swarrow \searrow \\ 2 \times \sqrt{3} \end{array} \\ & 8\sqrt{(2 + \sqrt{3})^2} = 8(2 + \sqrt{3}) \\ & = 16 + 8\sqrt{3} \\ & \sqrt{19 + 8\sqrt{3}} \\ & 2 \times 4\sqrt{3} = \sqrt{(4 + \sqrt{3})^2} = 4 + \sqrt{3} \\ & \begin{array}{c} 4 \times \sqrt{3} \end{array} \\ & \sqrt{-\sqrt{3} + 4 + \sqrt{3}} \\ & \sqrt{4} = 2 \end{aligned}$$

Q) Find the positive square root of $92 - 24\sqrt{6 - 4\sqrt{2}}$.

$92 - 24\sqrt{6 - 4\sqrt{2}}$ का धनात्मक वर्गमूल ज्ञात कीजिए।

$$\begin{array}{c} 2 \times 2\sqrt{2} \\ / \quad \backslash \\ 2 \times \sqrt{2} \end{array}$$

$$\sqrt{(2 - \sqrt{2})^2} = (2 - \sqrt{2})$$

$$92 - 24(2 - \sqrt{2})$$

$$92 - 48 + 24\sqrt{2}$$

$$44 + 24\sqrt{2}$$

$$\sqrt{44 + 24\sqrt{2}}$$

$$\begin{array}{c} 2 \times 12\sqrt{2} \\ / \quad \backslash \\ 6 \times 2\sqrt{2} \end{array}$$

$$\sqrt{(6 + 2\sqrt{2})^2}$$

$$= 6 + 2\sqrt{2}$$

Q) $\sqrt{28 + 10\sqrt{3}} - \sqrt{7 - 4\sqrt{3}}$ का मान किसके निकटतम है ?

$$\begin{array}{c} 2 \times 5\sqrt{3} \\ / \quad \backslash \\ 5 \times \sqrt{3} \end{array}$$

$$\begin{array}{c} 2 \times 2\sqrt{3} \\ / \quad \backslash \\ 2 \times \sqrt{3} \end{array}$$

$$\sqrt{(5 + \sqrt{3})^2} - \sqrt{(2 - \sqrt{3})^2}$$

$$5 + \sqrt{3} - (2 - \sqrt{3})$$

$$5 + \sqrt{3} - 2 + \sqrt{3}$$

$$3 + 2\sqrt{3} \Rightarrow 3 + 2 \times 1.73$$

$$3 + 3.46$$

$$= 6.46$$

$$= 6.5$$

Q) The value of $\sqrt{5 + 2\sqrt{6}} - \frac{1}{\sqrt{5 + 2\sqrt{6}}}$ is :

$$\sqrt{5 + 2\sqrt{6}} - \frac{1}{\sqrt{5 + 2\sqrt{6}}} \text{ का मान है ;}$$

$$\sqrt{5+2\sqrt{6}}$$

$$\swarrow \searrow$$

$$\sqrt{3} \times \sqrt{2}$$

$$\sqrt{(\sqrt{3}+\sqrt{2})^2}$$

$$= (\sqrt{3}+\sqrt{2})$$

$$(\sqrt{3}+\sqrt{2}) - \frac{1}{(\sqrt{3}+\sqrt{2})}$$

$$(\sqrt{3}+\sqrt{2}) - (\sqrt{3}-\sqrt{2})$$

$$\downarrow$$

$$\cancel{\sqrt{3}} + \sqrt{2} - \cancel{\sqrt{3}} + \sqrt{2}$$

$$= 2\sqrt{2}$$

$$Q) \sqrt{30+12\sqrt{6}} - \sqrt{4+2\sqrt{3}} - \sqrt{19-6\sqrt{2}} = ?$$

$$\begin{array}{ccc} 2 \times 6\sqrt{6} & \swarrow \searrow & 2 \times 3\sqrt{2} \\ \swarrow \searrow & \sqrt{3} \times 1 & \swarrow \searrow \\ 2 \times 3 \times \sqrt{6} & & 3\sqrt{2} \times 1 \\ \swarrow \searrow & & \\ 2\sqrt{3} & 3\sqrt{2} & \end{array}$$

$$(3\sqrt{2}+2\sqrt{3}) - (\sqrt{3}+1) - (3\sqrt{2}-1)$$

$$3\sqrt{2} + 2\sqrt{3} - \sqrt{3} - \cancel{1} - 3\sqrt{2} + \cancel{1}$$

$$2\sqrt{3} - \sqrt{3}$$

$$= \sqrt{3}$$

$$Q) \text{ If } x = 8 + 2\sqrt{15}, \text{ then find } \sqrt{x} + \frac{2}{\sqrt{x}}$$

$$\sqrt{x} = \sqrt{8+2\sqrt{15}}$$

$$\swarrow \searrow$$

$$\sqrt{5} \times \sqrt{3}$$

$$\sqrt{x} = \sqrt{(\sqrt{5}+\sqrt{3})^2}$$

$$= \sqrt{5} + \sqrt{3}$$

$$\sqrt{5} + \sqrt{3} + \frac{2}{\sqrt{5}+\sqrt{3}}$$

$$\sqrt{5} + \sqrt{3} + \frac{\cancel{2}(\sqrt{5}-\sqrt{3})}{\cancel{2}}$$

$$\sqrt{5} + \cancel{\sqrt{3}} + \sqrt{5} - \cancel{\sqrt{3}}$$

$$= 2\sqrt{5}$$

$$Q) \left(\frac{2}{\sqrt{6} + \sqrt{4}} + \frac{1}{\sqrt{7} + \sqrt{6}} + \frac{1}{\sqrt{8} - \sqrt{7}} + 2 - 2\sqrt{2} \right) = ?$$

$$\frac{2(\sqrt{6} - \sqrt{4})}{2} + \frac{\sqrt{7} - \sqrt{6}}{1} + \frac{\sqrt{8} + \sqrt{7}}{1} + 2 - 2\sqrt{2}$$

$$\frac{\cancel{\sqrt{6}} - \cancel{\sqrt{4}}}{2} + \sqrt{7} - \cancel{\sqrt{6}} + \sqrt{8} + \sqrt{7} + 2 - 2\sqrt{2}$$

$$= 2\sqrt{7}$$

$$Q) \text{ What is } \frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{2020} + \sqrt{2021}}$$

$$\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{2020} + \sqrt{2021}} \text{ किसके बराबर है?}$$

$$\frac{1}{\sqrt{2} + 1} + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{4} + \sqrt{3}} + \dots + \frac{1}{\sqrt{2021} + \sqrt{2020}}$$

$$\sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \dots + \sqrt{2021} - \sqrt{2020}$$

$$\sqrt{2021} - 1$$

$$= \sqrt{2021} - 1$$

$$Q) \frac{1}{\sqrt{4} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{6}} + \dots + \frac{1}{\sqrt{224} + \sqrt{225}} = ?$$

$$\frac{1}{\sqrt{5} + \sqrt{4}} + \frac{1}{\sqrt{6} + \sqrt{5}} + \dots + \frac{1}{\sqrt{225} + \sqrt{224}}$$

$$\sqrt{5} - \sqrt{4} + \sqrt{6} - \sqrt{5} + \sqrt{7} - \sqrt{6} + \dots + \sqrt{225} - \sqrt{224}$$

$$\sqrt{225} - \sqrt{4}$$

$$15 - 2 = 13$$

$$Q) \text{ What is the value of } \frac{1}{4 - \sqrt{15}} - \frac{1}{\sqrt{15} - \sqrt{14}} + \frac{1}{\sqrt{14} - \sqrt{13}} - \frac{1}{\sqrt{13} - \sqrt{12}} +$$

$$\frac{1}{\sqrt{12} - \sqrt{11}} - \frac{1}{\sqrt{11} - \sqrt{10}} + \frac{1}{\sqrt{10} - 3} - \frac{1}{3 - \sqrt{8}}$$

$$\frac{1}{\sqrt{16} - \sqrt{15}} - \frac{1}{\sqrt{15} - \sqrt{14}} + \frac{1}{\sqrt{14} - \sqrt{13}} - \frac{1}{\sqrt{13} - \sqrt{12}} + \frac{1}{\sqrt{12} - \sqrt{11}} - \frac{1}{\sqrt{11} - \sqrt{10}} +$$

$$\frac{1}{\sqrt{10} - 3} - \frac{1}{3 - \sqrt{8}} \text{ का मान क्या है?}$$

$$\begin{aligned} & \sqrt{16} + \sqrt{15} - (\sqrt{15} + \sqrt{14}) + \sqrt{14} + \sqrt{13} - (\sqrt{13} + \sqrt{12}) + \sqrt{12} + \sqrt{11} - (\sqrt{11} + \sqrt{10}) + \\ & \quad \sqrt{10} + \sqrt{9} - (\sqrt{9} + \sqrt{8}) \\ & \sqrt{16} + \cancel{\sqrt{15}} - \cancel{\sqrt{15}} - \cancel{\sqrt{14}} + \cancel{\sqrt{14}} + \sqrt{13} - \cancel{\sqrt{13}} - \cancel{\sqrt{12}} + \cancel{\sqrt{12}} + \sqrt{11} - \cancel{\sqrt{11}} - \cancel{\sqrt{10}} + \cancel{\sqrt{10}} \\ & \quad + \cancel{\sqrt{9}} - \cancel{\sqrt{9}} - \sqrt{8} \end{aligned}$$

$$\begin{aligned} & \sqrt{16} - \sqrt{8} \\ & = 4 - 2\sqrt{2} \end{aligned}$$

$$\textcircled{8} \quad \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{2}+1}{\sqrt{2}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{2}-1}{\sqrt{2}+1} = ?$$

$$\frac{(\sqrt{3}+1)(\sqrt{3}+1)}{3-1} + \frac{(\sqrt{2}+1)(\sqrt{2}+1)}{2-1} + \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{3-1} + \frac{(\sqrt{2}-1)(\sqrt{2}-1)}{2-1}$$

$$\frac{(\sqrt{3}+1)^2}{2} + \frac{(\sqrt{3}-1)^2}{2} + \frac{(\sqrt{2}+1)^2}{1} + \frac{(\sqrt{2}-1)^2}{1}$$

$$\frac{1}{2} (2(3+1) + 2(2+1))$$

$$\begin{aligned} & 4 + 2 \times 3 \\ & 4 + 6 = 10 \end{aligned}$$

$$\textcircled{9} \quad \left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \right)^2 + \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \right)^2 \text{ is equal to :}$$

$$\left[\frac{(\sqrt{5}+\sqrt{3})^2}{2} \right]^2 + \left[\frac{(\sqrt{5}-\sqrt{3})^2}{2} \right]^2$$

$$\frac{(8+2\sqrt{15})^2}{4} + \frac{(8-2\sqrt{15})^2}{4}$$

$$\frac{1}{4} [2(8^2 + (2\sqrt{15})^2)]$$

$$\frac{1}{2} [2 \times (64 + 60)]$$

$$\frac{1}{2} (124)$$

$$= 62$$

$$\textcircled{10} \quad (\sqrt{3} + \sqrt{2})^2 - (\sqrt{3} - \sqrt{2})^2 = ?$$

$$4 \times \sqrt{3} \times \sqrt{2}$$

$$= 4\sqrt{6}$$

$$(a+b)^2 - (a-b)^2 = 4ab$$

Q) If $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$, find $a+b$

$$\frac{(\sqrt{3}+\sqrt{2})^2}{1}$$

$$3+2+2\sqrt{6} = a+b\sqrt{6}$$

$$5+2\sqrt{6} = a+b\sqrt{6}$$

$$a=5 \quad b=2$$

$$a+b = 5+2 = 7$$

Q) $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} = ?$

$$\frac{\cancel{3}\sqrt{2}(\sqrt{6}-\sqrt{3})}{\cancel{3}} - \frac{\cancel{4}\sqrt{3}(\sqrt{6}-\sqrt{2})}{\cancel{4}} + \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{1}$$

$$\sqrt{12}-\sqrt{6} - (\sqrt{18}-\sqrt{6}) + \sqrt{18}-\sqrt{12}$$

$$\cancel{\sqrt{12}} - \sqrt{6} - \cancel{\sqrt{18}} + \sqrt{6} + \sqrt{18} - \cancel{\sqrt{12}}$$

$$= 0$$

1. What will be the positive square root of $19 + 4\sqrt{21}$

$19 + 4\sqrt{21}$ का धनात्मक वर्गमूल क्या होगा?

- (a) $\sqrt{7} + 2\sqrt{3}$
- (b) $\sqrt{3} + 2\sqrt{7}$
- (c) $\sqrt{2} + 3\sqrt{7}$
- (d) $\sqrt{7} + 3\sqrt{3}$

2. What is the square root of $14 + 6\sqrt{5}$

$14 + 6\sqrt{5}$ का धनात्मक वर्गमूल क्या है?

- (a) $3 + \sqrt{5}$
- (b) $3 - \sqrt{5}$
- (c) $5 + \sqrt{3}$
- (d) $5 - \sqrt{3}$

3. What is the value of

I. $(2 + \sqrt{2}) + \left(\frac{1}{2+\sqrt{2}}\right) + \left(\frac{1}{2-\sqrt{2}}\right) + (2 - \sqrt{2})$?

I. $(2 + \sqrt{2}) + \left(\frac{1}{2+\sqrt{2}}\right) + \left(\frac{1}{2-\sqrt{2}}\right) + (2 - \sqrt{2})$ का मान क्या है?

- (a) 2
- (b) 4
- (c) 8
- (d) 6

4. Simplify: $\frac{1}{\sqrt{100}-\sqrt{99}} - \frac{1}{\sqrt{99}-\sqrt{98}} + \frac{1}{\sqrt{98}-\sqrt{97}} - \frac{1}{\sqrt{97}-\sqrt{96}} + \dots + \frac{1}{\sqrt{2}-\sqrt{1}}$

- (a) 10
- (b) 9
- (c) 13
- (d) 11

5. $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}}\right) + \frac{\sqrt{3}-1}{\sqrt{3}+1} = ?$

- (a) 16
- (b) $13 - \sqrt{3}$
- (c) $16 - \sqrt{3}$
- (d) $16 + \sqrt{3}$

6. $\left(\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}}\right) = ?$

(a) 0

(b) 1

(c) 5

(d) $\frac{1}{3}$

7. $(3 + 2\sqrt{2})^2 + (3 - 2\sqrt{2})^2 = ?$

(a) 14

(b) 24

(c) 17

(d) 34

8. $\left(\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}\right)^2 + \left(\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}\right)^2 = ?$

(a) 142

(b) 140

(c) 71

(d) 60

9. If $x = 19 + 8\sqrt{3}$, then find $\sqrt{x} + \frac{13}{\sqrt{x}}$

(a) 38

(b) 8

(c) 19

(d) 16

10. If $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = a + b\sqrt{15}$, then find $a + b$

(a) 2

(b) 8

(c) 3

(d) 5

11 (If) यदि $x = \sqrt{-\sqrt{3} + \sqrt{3 + 8(2 + \sqrt{3})}}$ (find the value of x) तो x का

मान क्या होगा?

(a) 3 (b) 4

(c) 1 (d) 2

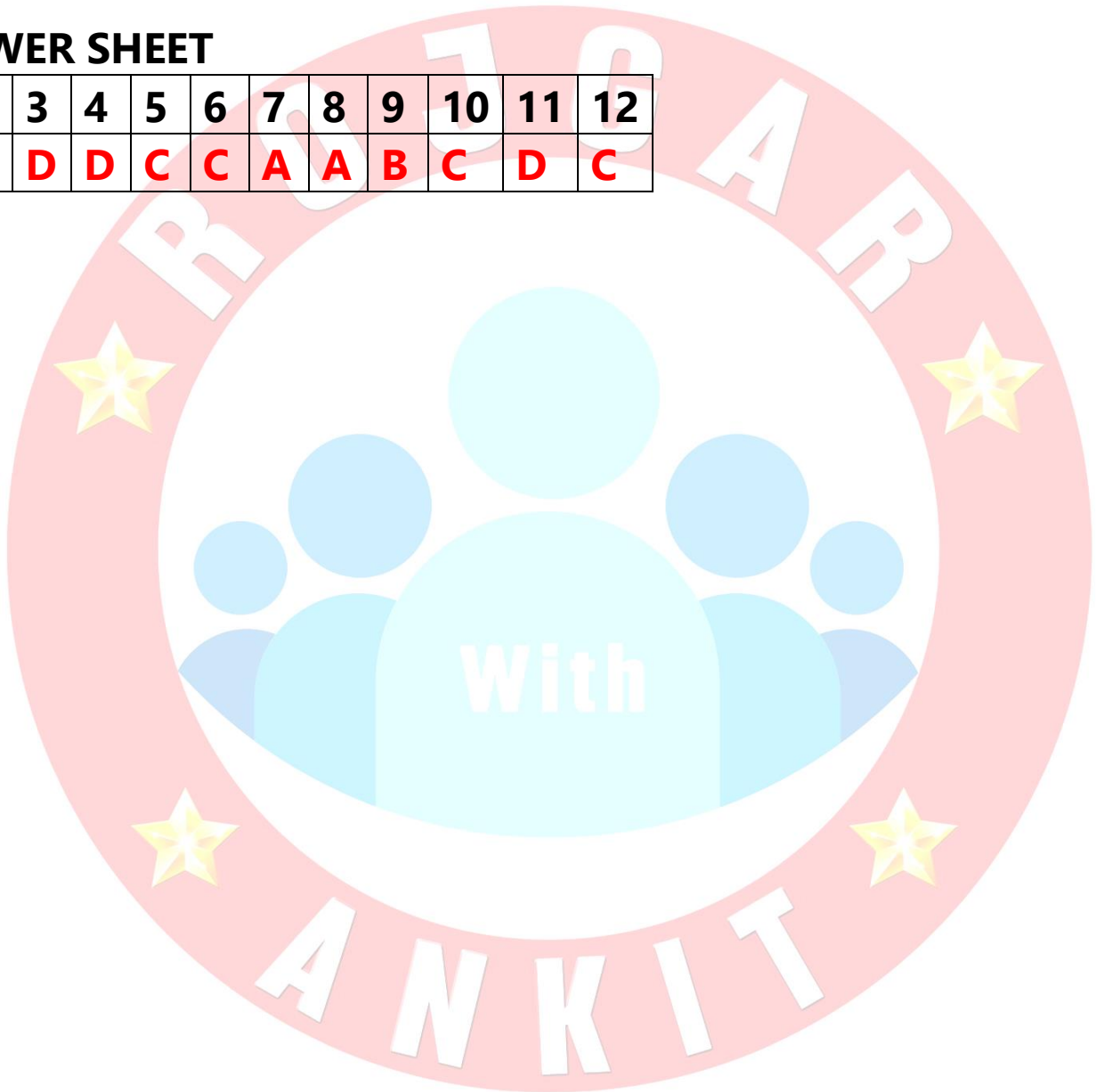
12. If $\sqrt{86 - 60\sqrt{2}} = a - b\sqrt{2}$, what is the correct value of $\sqrt{a^2 + b^2}$ upto one decimal place?

यदि $\sqrt{86 - 60\sqrt{2}} = a - b\sqrt{2}$ है तो $\sqrt{a^2 + b^2}$ का एक दशमलव तक शुद्ध मान क्या होगा?

- (a) 8.4
- (b) 8.2
- (c) 7.8
- (d) 7.2

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10	11	12
A	A	D	D	C	C	A	A	B	C	D	C



Worksheet solution

Sol 1

$$19 + 4\sqrt{21}$$

$$= 19 + 2\sqrt{4 \times 21}$$

$$= 7 + 12 + 2\sqrt{7 \times 12}$$

$$= (\sqrt{7})^2 + (\sqrt{12})^2 + 2\sqrt{7} \sqrt{12}$$

$$= (\sqrt{7} + \sqrt{12})^2$$

वर्ग मूल = $\sqrt{(\sqrt{7} + \sqrt{12})^2}$

$$= \sqrt{7} + 2\sqrt{3}$$

Sol 2

$$14 + 6\sqrt{5}$$

$$= \sqrt{14 + 2 \times 3\sqrt{5}}$$

$$= \sqrt{9 + 5 + 2 \times 3\sqrt{5}}$$

$$= \sqrt{(3)^2 + (\sqrt{5})^2 + 2 \times 3\sqrt{5}}$$

$$= \sqrt{(3 + \sqrt{5})^2}$$

$$= 3 + \sqrt{5}$$

Sol 3

$$(2 + \sqrt{2}) + \left(\frac{1}{2 + \sqrt{2}} \right) +$$

$$\left(\frac{1}{2 - \sqrt{2}} \right) + (2 - \sqrt{2})$$

$$= \frac{4 + 2\sqrt{2} + 2 - \sqrt{2} + 2 + \sqrt{2} + 4 - 2\sqrt{2}}{2}$$

$$= \frac{12}{2} = 6$$

Sol 4

$$\frac{1}{\sqrt{100} - \sqrt{99}} - \frac{1}{\sqrt{99} - \sqrt{98}} + \dots$$

$$\dots - \frac{1}{\sqrt{2} - \sqrt{1}}$$

$$\left[\frac{1}{\sqrt{100} - \sqrt{99}} \times \frac{\sqrt{100} + \sqrt{99}}{\sqrt{100} + \sqrt{99}} \right]$$
$$= \frac{\sqrt{100} + \sqrt{99}}{1}$$

$$= (\sqrt{100} + \sqrt{99}) - (\sqrt{99} + \sqrt{98})$$

$$\dots - (\sqrt{2} + \sqrt{1})$$

$$= \sqrt{100} + \sqrt{1}$$

$$= 10 + 1 = 11 \text{ ans}$$

Sol 5

$$\left(\frac{2 + \sqrt{3}}{2 - \sqrt{3}} + \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \right) + \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \left[\frac{(4+2\sqrt{3}+2\sqrt{3}+3) + (4-2\sqrt{3}-2\sqrt{3}+3)}{(2-\sqrt{3})(2+\sqrt{3})} \right] + \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \left[\frac{7+4\sqrt{3}+7-4\sqrt{3}}{1} \right] + \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= 14 + \frac{(\sqrt{3}-1)^2}{2}$$

$$= 14 + 2 - \sqrt{3} = 16 - \sqrt{3}$$

Sol 6

$$\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{\sqrt{9}-\sqrt{8}} \times \frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}} = \frac{\sqrt{9}+\sqrt{8}}{1}$$

$$= (\sqrt{9}+\sqrt{8})$$

similarly ; $\frac{1}{(\sqrt{8}-\sqrt{7})} = (\sqrt{8}+\sqrt{7})$, $\frac{1}{(\sqrt{7}-\sqrt{6})} = (\sqrt{7}+\sqrt{6})$

$$\frac{1}{(\sqrt{6}-\sqrt{5})} = (\sqrt{6}+\sqrt{5}) \quad \frac{1}{(\sqrt{5}-\sqrt{4})} = \sqrt{5}+\sqrt{4}$$

Given expression = $(\sqrt{9}+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+\sqrt{4})$

$$= 5(\sqrt{9}+\sqrt{4}) = (3+2) = 5 \underline{\text{Ans}}$$

Sol 7

$$\begin{aligned}
 & (3+2\sqrt{2})^2 + (3-2\sqrt{2})^2 \\
 &= 7-4\sqrt{2} + 7+4\sqrt{2} \\
 &= 14
 \end{aligned}$$

Sol 8

$$\begin{aligned}
 & \left[\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} \right]^2 + \left[\frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}} \right]^2 \\
 &= \frac{12 + 2\sqrt{35}}{12 - 2\sqrt{35}} + \frac{12 - 2\sqrt{35}}{12 + 2\sqrt{35}} \\
 &= \frac{284 + 284}{4} = 142
 \end{aligned}$$

Sol 9

$$\begin{aligned}
 x &= 19 + 8\sqrt{3} \\
 x &= 16 + 3 + 8\sqrt{3} \\
 x &= (4 + \sqrt{3})^2 \\
 \sqrt{x} &= \sqrt{(4 + \sqrt{3})^2} \\
 \sqrt{x} &= 4 + \sqrt{3}
 \end{aligned}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}}$$

$$\frac{1}{\sqrt{x}} = \frac{4 - \sqrt{3}}{13}$$

$$\frac{13}{\sqrt{x}} = 4 - \sqrt{3}$$

$$\begin{aligned}
 \sqrt{x} + \frac{13}{\sqrt{x}} &= 4 + \sqrt{3} + 4 - \sqrt{3} \\
 &= 8
 \end{aligned}$$

Sol 10

$$\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = a + b\sqrt{5}$$

$$\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$= a + b\sqrt{5}$$

$$\frac{8 - 2\sqrt{15}}{2} = a + b\sqrt{5}$$

$$\therefore a = 4, b = -1$$

$$4 - \sqrt{5} = 4 - \sqrt{5}$$

$$\therefore a + b = 4 - 1 = 3$$

Sol 11

$$\begin{aligned}
 x &= \sqrt{-\sqrt{3} + \sqrt{3 + 8(2 + \sqrt{3})}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{3 + 16 + 8\sqrt{3}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{19 + 8\sqrt{3}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{(4)^2 + (\sqrt{3})^2 + 2 \cdot 4 \cdot \sqrt{3}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{(4 + \sqrt{3})^2}} \\
 &= \sqrt{-\sqrt{3} + 4 + \sqrt{3}} = 2
 \end{aligned}$$

Sol 12

$$\sqrt{86 - 60\sqrt{2}} = a - b\sqrt{2}$$

$$\sqrt{50 + 36 - 2 \times 6 \times 5\sqrt{2}} = a - b\sqrt{2}$$

$$\sqrt{(5\sqrt{2} - 6)^2} = a - b\sqrt{2}$$

$$a - b\sqrt{2} = -6 + 5\sqrt{2}$$

$$a = -6 \text{ and } b = -5$$

$$\sqrt{(-6)^2 + (-5)^2} = \sqrt{36 + 25}$$

$$= \sqrt{61} = 7.8$$

QURDS AND INDICES

Q) $(\sqrt{5}+\sqrt{4})$, $(\sqrt{7}+\sqrt{2})$, $(\sqrt{6}+\sqrt{3})$ find the value of largest and smallest number.

$(\sqrt{5}+\sqrt{4})$, $(\sqrt{7}+\sqrt{2})$, $(\sqrt{6}+\sqrt{3})$ सबसे बड़ी और सबसे छोटी संख्या का मान ज्ञात करें।

$$\underbrace{(\sqrt{5}+\sqrt{4})}_{20}, \underbrace{(\sqrt{7}+\sqrt{2})}_{14}, \underbrace{(\sqrt{6}+\sqrt{3})}_{18}$$

$$\sqrt{5}+\sqrt{4}, \sqrt{7}+\sqrt{2}$$

Q) What is the value of $\frac{2\sqrt{10}}{\sqrt{5}+\sqrt{2}-\sqrt{7}} - \sqrt{\frac{5-2}{5+2}} - \frac{3}{\sqrt{7}-2}$

$$\frac{2\sqrt{10}}{\sqrt{5}+\sqrt{2}-\sqrt{7}} - \sqrt{\frac{5-2}{5+2}} - \frac{3}{\sqrt{7}-2} \text{ का मान क्या है ?}$$

$$\frac{2\sqrt{10}}{\sqrt{5}+\sqrt{2}-\sqrt{7}} \times \frac{\sqrt{5}+\sqrt{2}+\sqrt{7}}{\sqrt{5}+\sqrt{2}+\sqrt{7}} - \sqrt{\frac{5-2}{5+2} \times \frac{5-2}{5-2}} - \frac{3}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$$

$$\frac{2\sqrt{10}(\sqrt{5}+\sqrt{2}+\sqrt{7})}{(\sqrt{5}+\sqrt{2})^2 - (\sqrt{7})^2}$$

$$\sqrt{\frac{(5-2)^2}{5-4}}$$

$$\frac{3(\sqrt{7}+2)}{7-4}$$

$$\frac{2\sqrt{10}(\sqrt{5}+\sqrt{2}+\sqrt{7})}{7+2\sqrt{10}-7}$$

$$(\sqrt{5}-2)$$

$$(\sqrt{7}+2)$$

$$\sqrt{5}+\sqrt{2}+\sqrt{7} - \sqrt{5} + 2 - \sqrt{7} - 2$$

$$= \sqrt{2}$$

TYPE-III

Q) $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = (a+b) + (a-b)\sqrt{35}$ then find $(a-b)$

$$\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$$

$$\frac{(\sqrt{7}+\sqrt{5})^2}{7-5}$$

$$\frac{7+5+2\sqrt{35}}{2}$$

$$\frac{12+2\sqrt{35}}{2}$$

$$\frac{2(6+\sqrt{35})}{2} = 6+\sqrt{35}$$

$$a+1=6$$

$$\boxed{a=5}$$

$$a+b=1$$

$$5+b=1$$

$$\boxed{b=-4}$$

$$a-b$$

$$5-(-4)$$

$$=9$$

Q) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = (a+b) + (b+1)\sqrt{3}$ find $a-b$

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{(2+\sqrt{3})^2}{4-3}$$

$$4+3+4\sqrt{3}$$

$$(7+4\sqrt{3})$$

$$a+b=7$$

$$a+3=7$$

$$\boxed{a=4}$$

$$b+1=4$$

$$\boxed{b=3}$$

$$a-b$$

$$4-3$$

$$=1$$

TYPE-IV

$x = \sqrt{a\sqrt{a\sqrt{a\cdots}}} \text{ n बार}$

Ans: $a^{\frac{2^n-1}{2^n}}$

Q $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$ (n=4)

$$\boxed{a=2}$$

$$2^{\frac{2^4-1}{2^4}} = 2^{\frac{16-1}{16}}$$

$$= 2^{\frac{15}{16}}$$

Q) $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}}$ Simplify the / को सरल करें।

$$\begin{aligned} n &= 5 \\ a &= 7 \\ 7^{\frac{2^5-1}{2^5}} \\ 7^{\frac{32-1}{32}} &= 7^{\frac{31}{32}} \end{aligned}$$

$x = \sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}} \dots \infty$

$$\begin{aligned} x &= \sqrt{ax} \\ x^2 &= ax \\ x &= a \end{aligned}$$

Q) $\sqrt{x\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}} \dots \infty = ?$

Q) $\sqrt[5]{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}}}} \dots \infty ?$

Q) If $x^m = \sqrt[14]{x\sqrt{x\sqrt{x}}}$, then what is the value of m ?

यदि $x^m = \sqrt[14]{x\sqrt{x\sqrt{x}}}$ तो m का मान क्या है?

$$\begin{aligned} \frac{(x^m)^{14}}{x^{14m}} &= \frac{x\sqrt{x\sqrt{x}}}{x\sqrt{x^1 x^{1/2}} \\ &= \frac{x\sqrt{x^{3/2}}}{x(x^{3/2})^{1/2}} \\ &= \frac{x^1 \times x^{3/4}}{x^{1+\frac{3}{4}}} \\ &= \boxed{x^{7/4}} \\ x^{14m} &= x^{7/4} \\ 14m &= \frac{7}{4} \\ m &= \frac{1}{8} \end{aligned}$$

$$\textcircled{Q} x = \sqrt[3]{a \sqrt{b \sqrt[3]{a \sqrt{b \sqrt[3]{a \sqrt{b \dots \infty}}}}} = ?$$

$$x = \sqrt[3]{a \sqrt{bx}}$$

$$x^3 = a \sqrt{bx}$$

$$(x^3)^2 = a^2 \times bx$$

$$x^6 = a^2 bx$$

$$x^5 = a^2 b$$

$$x = (a^2 b)^{\frac{1}{5}}$$

$$\textcircled{Q} \sqrt[3]{\overset{a}{4} \sqrt{\underset{b}{2} \sqrt[3]{4 \sqrt{2 \sqrt[3]{4 \dots \infty}}}}} = ?$$

$$(a^2 b)^{1/5}$$

$$(4^2 \times 2)^{1/5}$$

$$(32)^{1/5}$$

$$(2^5)^{1/5}$$

$$= 2$$

$$\textcircled{Q} \sqrt[3]{\underset{a}{49} \sqrt{\underset{b}{7} \sqrt[3]{49 \sqrt{7 \sqrt[3]{49 \dots \infty}}}}} = ?$$

$$(a^2 b)^{1/5}$$

$$(49^2 \times 7)^{1/5}$$

$$((7^2)^2 \times 7^1)^{1/5}$$

$$(7^4 \times 7^1)^{1/5}$$

$$(7^5)^{1/5}$$

$$= 7$$

$$\# \sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a + \dots \infty}}}} = \frac{1 + \sqrt{1 + 4a}}{2}$$

$$\# \sqrt{a - \sqrt{a - \sqrt{a - \sqrt{a - \dots \infty}}}} = \frac{-1 + \sqrt{1 + 4a}}{2}$$

Q) $\sqrt{7 + \sqrt{7 + \sqrt{7 + \dots \infty}}}$

$$\frac{1 + \sqrt{1 + 4 \times 7}}{2}$$

$$\frac{1 + \sqrt{29}}{2}$$

Q) $\sqrt{30 + \sqrt{30 + \sqrt{30 + \sqrt{30 + \dots \infty}}}}$ Find the value of $\sqrt{\quad}$ का मान ज्ञात करें।

$$\frac{1 + \sqrt{1 + 4a}}{2}$$

$$\frac{1 + \sqrt{1 + 4 \times 30}}{2} = \frac{1 + \sqrt{121}}{2}$$

$$\frac{1 + 11}{2} = \frac{12}{2} = 6$$

Ind

30
5 × 6
diff = 1

Ans 6

• ऐसी 2 संख्या से factorization करना है जिनका diff 1 हो।

• अगर बीच में (+) का sign है तो Ans बड़ी संख्या होगा और (-) का sign है तो Ans छोटी संख्या होगा।

Q) If $x^2 - 20 = \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \dots \infty}}}}$ Infinite terms then what is x equal to?

4 × 5

$$x^2 - 20 = 5$$

$$x^2 = 25$$

$$x = \sqrt{25}$$

$$= 5$$

Q) Find the value of the given expression.

दिए गए व्यंजक का मान ज्ञात कीजिए।

$$\sqrt{20 - \sqrt{20 - \sqrt{20 - \sqrt{20 - \dots \infty}}}}$$

4 × 5
4

Q) The value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$ is
 $\sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}$ का मान है।

$$\frac{1 + \sqrt{1 + 4 \times 1}}{2}$$

$$\frac{1 + \sqrt{5}}{2}$$

$$\frac{1 + 2.2}{2}$$

$$\sim \frac{3.2}{2} = 1.6$$

1 और 2 के बीच है Lies between 1 and 2

Q) $\sqrt{42 - \sqrt{42 - \sqrt{42 - \sqrt{42 - \dots}}}} = ?$

6 $\times 7$

Ans = 6

Q) If $x = \sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a + \dots}}}}$ and $y = \sqrt{8 - \sqrt{8 - \sqrt{8 - \sqrt{8 - \dots}}}}$
 then find $x - y$?

$$x = \frac{1 + \sqrt{1 + 4a}}{2}$$

$$y = \frac{-1 + \sqrt{1 + 4a}}{2}$$

$$x - y = \frac{1 + \sqrt{1 + 4a}}{2} - \left[\frac{-1 + \sqrt{1 + 4a}}{2} \right]$$

$$\frac{1 + \sqrt{1 + 4a} + 1 - \sqrt{1 + 4a}}{2} = \frac{2}{2} = 1$$

Q) If $\sqrt{2\sqrt{4\sqrt{2\sqrt{4\sqrt{2}}}}} = 32^a$, then find the value of a
 यदि $\sqrt{2\sqrt{4\sqrt{2\sqrt{4\sqrt{2}}}}} = 32^a$, तो a का मान ज्ञात करें।

$$(2^5)^a$$

$$2^{5a}$$

$$2^2 \times 2^{1/2}$$

$$2 \left(2^{\frac{5}{2}} \right)^{1/2}$$

$$2^1 \times 2^{\frac{5}{4}}$$

$$2^2 \left(2^{9/4} \right)^{1/2}$$

$$2^2 \times 2^{9/8}$$

$$2^1 \left(2^{\frac{25}{8}} \right)^{1/2}$$

$$2^1 \times 2^{\frac{25}{16}}$$

$$\left(2^{\frac{41}{16}} \right)^{1/2}$$

$$\left(2^{\frac{41}{32}} \right) = 2^{5a}$$

$$5a = \frac{41}{32}$$

$$a = \frac{41}{160}$$

1. $\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}} = ?$

(a) $5^{\frac{1}{4}}$

(b) $5^{\frac{3}{4}}$

(c) $5^{\frac{7}{8}}$

(d) $5^{\frac{15}{16}}$

2. $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}} \dots \infty = ?$

(a) 0

(b) $7^{\frac{1}{2}}$

(c) 7

(d) 1

3. $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots \infty}}}} = ?$

(a) 4

(b) 2

(c) 3

(d) 8

4. Solve $(\sqrt{42 + \sqrt{42 + \sqrt{42 + \dots \infty}}}) \times (\sqrt{42 - \sqrt{42 - \sqrt{42 - \dots \infty}}})$

(a) 48

(b) 40

(c) 42

None of these

5. $\sqrt[3]{a^5 \sqrt{b^{10} \sqrt[3]{a^5 \sqrt{b^{10} \sqrt{a^5 \dots \infty}}}}} = ?$

(a) $(ab^2)^{\frac{1}{5}}$

(b) $(a^2b)^{\frac{1}{5}}$

(c) a^2b^2

(d) $(a^2b^2)^{\frac{1}{10}}$

6. $\sqrt{2 - \sqrt{2 - \sqrt{2 - \sqrt{2 - \sqrt{2 - \dots - \infty}}}}} = ?$

- (a) 2
(b) 0
(c) 1
(d) $2\sqrt{2}$

7. $\frac{\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}}}{\sqrt[3]{8}} = ?$

- (a) 8
(b) 4
(c) $1/2$
(d) 2

8. $\sqrt{74 + \sqrt{700 \times \sqrt{42 + \sqrt{42 + \sqrt{42 \dots \dots \infty}}}}} = ?$

- (a) 11
(b) 12
(c) 10
(d) 13

9. $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b \cdot \sqrt[3]{2} + c$, find $a + b + c$

- (a) 3
(b) 2
(c) 0
(d) 1

10. $\sqrt{12 + 2\sqrt{6} + 2\sqrt{21} + 2\sqrt{14}} = ?$

- (a) $(\sqrt{7} + \sqrt{3} + \sqrt{2})$
(b) $(\sqrt{5} + \sqrt{2} + \sqrt{1})$
(c) $(\sqrt{7} - \sqrt{3} + \sqrt{2})$
(d) $(\sqrt{5} - \sqrt{2} + \sqrt{1})$

11. $\left(\frac{m^2}{n^2}\right)^{4x+5} = \left(\frac{n^3}{m^3}\right)^{\frac{x}{3}+6}$ find $x = ?$

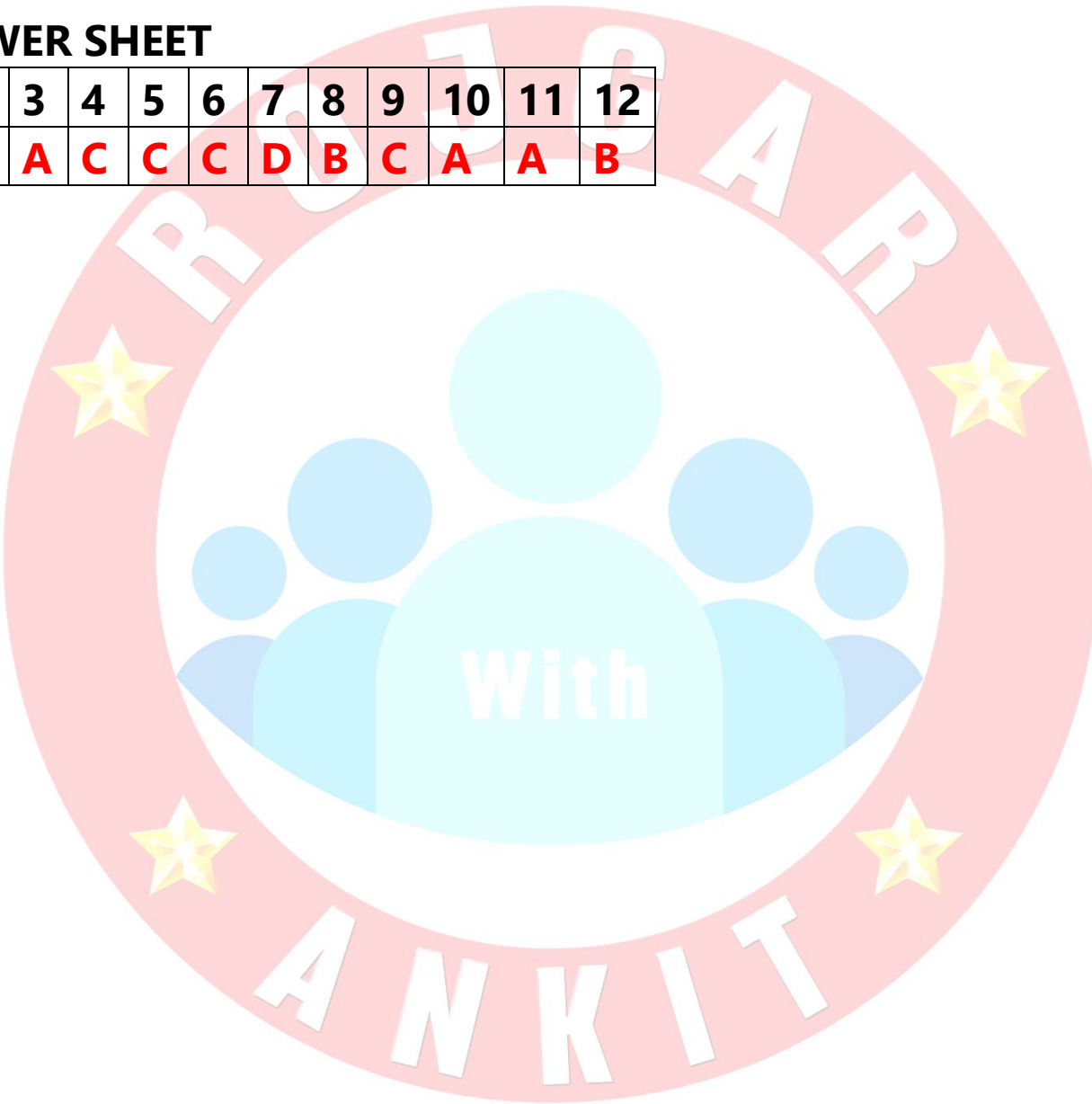
- (a) $\frac{-28}{9}$ (b) $\frac{-28}{7}$
(c) $\frac{28}{7}$ (d) $\frac{+28}{9}$

12. $\sqrt[4]{\sqrt[10]{\left[\sqrt[12]{\left(\sqrt[4]{3^2}\right)^6}\right]^8}}^5} = ?$

- (a) $\sqrt[3]{3}$
- (b) $\sqrt[4]{3}$
- (c) $\sqrt{3}$
- (d) 9

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10	11	12
D	C	A	C	C	C	D	B	C	A	A	B



Sol 9

$$\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$$

$$= \sqrt[3]{2} - 1 = a\sqrt[3]{4} + b\sqrt[3]{2} + c$$

$$a = 0, b = 1, c = -1$$

$$\therefore a + b + c = 0 + 1 - 1 = 0$$

Sol 10

$$\sqrt{12 + 2\sqrt{6} + 2\sqrt{21} + 2\sqrt{14}}$$

$$= \sqrt{(\sqrt{7})^2 + (\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3} \times \sqrt{2} + 2 \times \sqrt{7} \times \sqrt{3} + 2\sqrt{7} \times \sqrt{2}}$$

$$= \sqrt{7} + \sqrt{3} + \sqrt{2}$$

Sol 11

$$\left(\frac{m^2}{n^2}\right)^{4n+5} = \left(\frac{n^3}{m^3}\right)^{\frac{n}{3}+6}$$

$$\left(\frac{m}{n}\right)^{8n+10} = \left(\frac{n}{m}\right)^{-n-18}$$

$$8n+10 = -n-18$$

$$n = -\frac{28}{9}$$

Sol 12

$$4 \sqrt[10]{\left[2 \sqrt[12]{\left[4 \sqrt[3]{3^2}\right]^6}\right]^8}^5$$

$$= \sqrt[3]{\frac{2 \times 6 \times 8 \times 5}{4 \times 10 \times 12 \times 4}} = 3^{\frac{1}{4}} = \sqrt[4]{3}$$